Reading: Ireland and Rosen, Chapter 5 (including the exercises) and into Chapter 6

Problems:

1. (Don’t turn this in, but make sure to do it if you haven’t yet.) Confirm for odd primes \( p \) that

\[
(-1)^{(p-1)/2} = \begin{cases} 
1 & \text{if } p \equiv 1 \pmod{4}, \\
-1 & \text{if } p \equiv 3 \pmod{4},
\end{cases}
\]

and

\[
(-1)^{(p^2-1)/8} = \begin{cases} 
1 & \text{if } p \equiv \pm 1 \pmod{8}, \\
-1 & \text{if } p \equiv \pm 3 \pmod{8}.
\end{cases}
\]

2. Use Quadratic Reciprocity to prove Euler’s observation by justifying the following computation for distinct odd primes \( p \) and \( q \) with \( 4q > p \):

\[
\left(\frac{q}{4q \pm p}\right) = (-1)^{\frac{q-1}{2}} 4q^{\frac{q+p-1}{2}} \left(\frac{4q \pm p}{q}\right) = (-1)^{\frac{q-1}{2} \frac{p-1}{2}} \left(\frac{\pm p}{q}\right)
\]

\[
= (-1)^{\frac{q-1}{2} \frac{p-1}{2}} \left(\frac{p}{q}\right) = \left(\frac{q}{p}\right).
\]

3. Read Ireland and Rosen Exercises 5.3; 5.4–5.8; 5.9, 5.10; 5.11; 5.12, 5.13; 5.14, 5.15; 5.16; 5.17 (just the Proposition); 5.18–5.21; 5.22; 5.23, 5.24; 5.38, and work a small selection of them.

4. Work Ireland and Rosen Exercises 5.25–5.28 and/or 5.29–5.31.