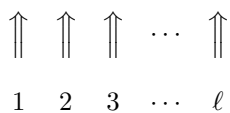


THE HAT PROBLEM

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Let ℓ be any positive integer. You might take it to be 20 or 39, or something. Imagine ℓ prisoners standing in the yard single file, one behind the other:



By the way, the prisoners are all men, since women behave better than men. Each of the prisoners can see the prisoners standing in front of him but not the ones standing behind him. That is, for any integers j and k :

$$1 \leq j < k \leq \ell$$

prisoner j can see prisoner k but prisoner k cannot see prisoner j . The dreaded warden strides into the yard. From a bag containing ℓ red hats and ℓ blue hats, he selects ℓ hats at random, one at a time. His assistant places the first hat on the head of prisoner 1, the second hat on the head of prisoner 2, and so forth. Now all the prisoners are wearing hats (red or blue). Each of the prisoners can see the hats on the heads of the prisoners standing in front of him but not the hat on his own head or the hats on the heads of those behind him. The warden tells the first prisoner to speak one word, either "red" or "blue." If he speaks the word which matches the color of his hat then the assistant immediately sets him free. If not then the assistant immediately shoots him. Next, the warden tells the second prisoner to speak one word, either "red" or "blue." If he speaks the word which matches the color of his hat then the assistant immediately sets him free. If not then the assistant immediately shoots him. And so forth. However, unknown to the warden and his assistant, the prisoners were aware, earlier, of the warden's depraved game. Of course, they had no way to know what would be the order in which they would be forced to stand or what would be the colors of their hats. Nevertheless, before coming to the yard, they planned a strategy. Prisoner j would look to the hats in front of him. When told to speak, he would say "red" if the number of blue hats in front of him was even; but would say "blue" if that number was odd. Unfortunately for prisoner 1, he would have only a fifty/fifty chance to go free. However, all the others could call out the correct colors of their hats, one by one in succession. They would all go free. Please explain.

Love, **Dad**