structure

reactions

statics

dynamics

$F_N$

$m$

$m_g$

$m_1$ $m_2$
FIG. 5. (Color online) Shows the nuclei that have neutron capture rates which effect a $F \approx 5\%$ or more abundance change for a rate increase or decrease of a factor of 10 (dark color squares), a factor of 50 (medium color squares), and a factor of 100 to 1000 (light color squares).
Nuclear reactions

• Basics of nuclear reactions
• Elastic scattering
• Optical models
• (DWBA)
Nuclear reactions

• Basics of nuclear reactions
• Elastic scattering
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• (DWBA)
A + B $\rightarrow$ D + C

Before reaction:

- Projectile B
- Target A

After reaction:

- Target-like fragment D
- Projectile-like fragment C

Reaction notation:

$$A(B, C)D$$
projec-le
B

before reaction

target A

reaction notation
B(A,D)C
A + B → D + C

after reaction

target-like fragment D

projectile-like fragment C
Before reaction:

projectile B

Target A

Reaction notation:

B(A, D)C

A + B → D + C

After reaction:

Interesting(boring, boring-like) interesting-like

Boring(interesting, interesting-like) boring-like

Projectile(target, target-like) projectile-like

Target(projectile, projectile-like) target-like
$N_{Z,A}(\theta, E)$
neutron/proton numbers

parity

angular momentum

linear momentum

total energy
Know:
• $p_a$
• $m_a$
• $m_b$
• $m_X$

Measure:
• $p_b$
• $\theta$

Use:
• Energy conservation
• Momentum conservation
$X(a,b)Y$

Kinematics for $^{95}\text{Sr}$ on $\text{CD}_2$

$\vec{p}_a \rightarrow \vec{p}_X = 0 \rightarrow \vec{p}_b$

$^{95}\text{Sr}(d,p)$

$^{95}\text{Sr}(p,p)$

$^{95}\text{Sr}(d,d)$

Steffen Cruz, 2015
\[
\mathbf{E}_{\text{beam}} = E_a
\]

\[
E = \frac{m_X}{m_a + m_X} E_{\text{beam}}
\]
Q-Value

how much energy you get by executing a process
Primary observable: cross-section

...kinda.

https://en.wikipedia.org/wiki/Cross_section_(geometry)
1 barn \equiv 1 \times 10^{-28} \text{ m}^2

^{208}\text{Pb}

r \approx 6 \text{ fm}

A \approx 1.1 \times 10^{-28} \text{ m}^2
Reaction classifications

Direct

Compound

http://queenofthedistracted.blogspot.ca/2010_10_01_archive.html
Reaction classifications

**Direct**
- one-step reaction
- final states keep “memory” of initial state
- few-body problem
- small momentum transfer
- short timescale ($10^{-22} s$)
- forward-peaked distribution
- high energy

**Compound**
- many step reaction
- all nucleons involved
- long timescale
- isotropic angular distribution
- low energy

http://queenofthedistracted.blogspot.ca/2010_10_01_archive.html
Nuclear reactions

- Basics of nuclear reactions
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plane wave

$V(\vec{r'})$
plane wave

$V(\mathbf{r})$
Potential assumptions:
- spherical potential
- short-range potential

Consequences:
- cylindrical symmetry
- angular momentum conservation
- asymptotic region
\[ \Psi_{asympt}(r, \theta) = e^{ikz} + \frac{f(\theta) e^{ikr}}{r} \]

scattering amplitude

Incoming plane wave \( \exp(ikz) \)

Outgoing spherical waves \( \exp(ikR)/R \)

\[ \frac{d\sigma}{d\Omega} = |f(\theta)|^2 \]
\[ \nabla^2 \Psi + \frac{2m}{\hbar} \left[ E - V(r) \right] \Psi = 0 \]

kinetic energy

potential energy
\[ \nabla^2 \Psi + \frac{2m}{\hbar} [E - V(r)] \Psi = 0 \]

TISE

separate angular and radial components

\[ \Psi = \sum_{\ell} a_\ell P_\ell(\cos \theta) \frac{u_\ell}{kr} \]

partial wave expansion of \( \Psi \)

Legendre polynomials
\[ \Psi = \sum_{\ell} a_\ell P_\ell(\cos \theta) \frac{u_\ell}{k r} \]

\[ \frac{d^2 u}{dr^2} + \frac{2m}{\hbar^2} \left[ E - V(r) - \frac{\hbar^2}{2m} \frac{\ell(\ell + 1)}{r^2} \right] u = 0 \]

Partial wave expansion of \( \Psi \)

Radial component of TISE

Angular momentum term
numerical solver

radial component of TISE

$V(r)$

boundary conditions

partial wave expansion of $\Psi$

$u_\ell(r)$
\[ \Psi = \sum_{\ell} a_\ell P_\ell(\cos \theta) \frac{u_\ell}{kr} \]

\[ \Psi_{asympt}(r, \theta) = e^{ikz} + f(\theta) \frac{e^{ikr}}{r} \]
\[ \Psi_{asymp}(r, \theta) = e^{ikz} + f(\theta) \frac{e^{ikr}}{r} \]

\[ e^{ikz} \sim \sum_{\ell=0}^{\infty} \frac{1}{2i} (2\ell + 1) i^\ell P_\ell(\cos \theta) \]

\[ \Psi = \sum_{\ell} a_\ell P_\ell(\cos \theta) \frac{u_\ell}{kr} \]
\[ \Psi \sim \sum_{\ell=0}^{\infty} \frac{1}{2i} (2\ell + 1) i^{\ell} P_\ell(\cos \theta) S_\ell e^{i(kr - \frac{\ell \pi}{2})} - e^{-i(kr - \frac{\ell \pi}{2})} \]

**S-matrix** indicates how many particles are in this outgoing channel.

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<th>If ( V(r) ) is...</th>
<th>0</th>
<th>real</th>
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<tr>
<td>then...</td>
<td>( S_\ell = 1 )</td>
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</table>
\[ \Psi \sim \sum_{\ell=0}^{\infty} \frac{1}{2i} (2\ell + 1) i^\ell P_\ell(\cos \theta) \left[ \frac{S_\ell e^{i(kr - \frac{\ell \pi}{2})} - e^{-i(kr - \frac{\ell \pi}{2})}}{kr} \right] \]

\[ \Psi_{\text{asym}}(r, \theta) = e^{ikz} + f(\theta) \frac{e^{ikr}}{r} \]
\[ f(\theta) = \frac{1}{2i\kappa} \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\cos \theta)(S_{\ell} - 1) \]
\[ f(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos \theta)(S_\ell - 1) \]

\[ S_\ell = e^{2i\delta_\ell} \]

\[ f(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos \theta)e^{i\delta_\ell} \sin \delta_\ell \]

phase shift
Potential assumptions:
- spherical potential
- short-range potential
plane wave

\[ V(r) \]
\[
\frac{\sigma}{\sigma_{Ruth}} \equiv \left| \frac{f_c(\theta) + f_n(\theta)}{|f_c(\theta)|^2} \right|^2
\]
Nuclear reactions

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Woods-Saxon central potential

Coulomb potential

spin-orbit
Woods-Saxon central potential

surface

spin-orbit
Use nuclear structure as a tool

Fit directly to data

Krane, Figure 3.4
Thompson and Nunes, Figures 5.2, 5.3
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Neutron and proton scattering

A = 24 - 209

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