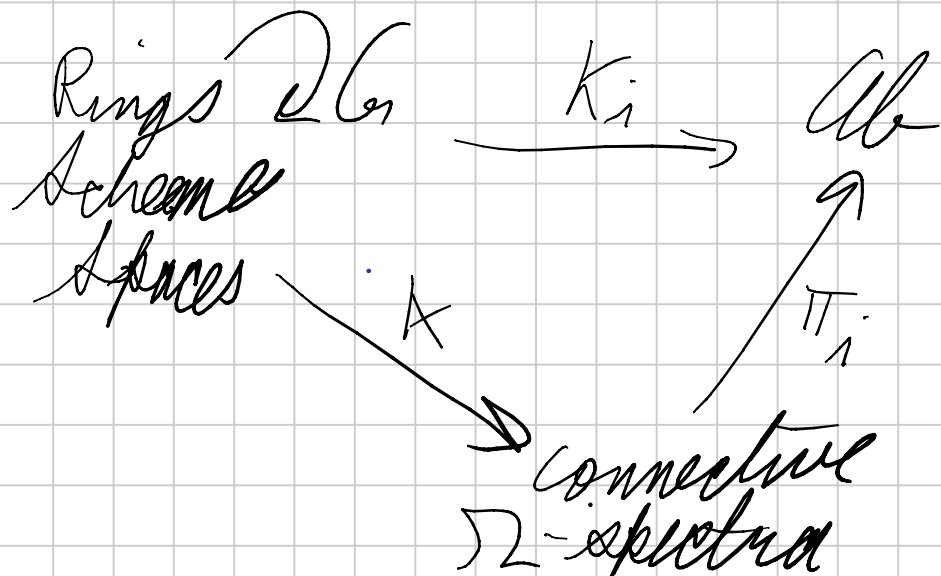


# Merling - Equivariant algebraic K-theory

Note Title

5/30/2015



We want  
 $KR_G \cong \Omega^V X_V$  for  
 $G$ -spectrum  $X$

If  $G$  acts on a ring  $R$ , it acts on  $R$ -modules  
via scalars

Given a map  $R \rightarrow S$  of  $G$ -action leads to

$\text{Mod}(R) \xrightarrow{\otimes_R S} \text{Mod}(S)$  not a  $G$ -module

$(gM) \otimes_R S \neq g(M \otimes_R S)$  but they are  
isomorphic  $\downarrow$  fin gen proj  $R$ -modules

Recall  $KR = \text{gp completion of } B(\text{mod } P(R))$

$P(R)$  is a symm monoidal category.

It leads to  $\Omega$ -spectra via May operad machine

or Segal's  $\Gamma$ -space machine

Then agree by May-Thomason theorem.

Equivalent analogs:

operad machine

$\Gamma$ -spaces

Guillou-May

Shimokawa 1980  $\rho$

May-Thomason theorem proved by  
 May-Merling-Osoyuki.

The input is a "genuine" symmetric monoidal category  
 with  $G$ -action, which I will explain.

Assume  $G$  is finite. What does "genuine" mean?

transfers?  $\mathcal{C}^H \xleftarrow{\text{res}} \mathcal{C}^G$   
 $\xrightarrow{\text{tr}}$

$j = |G/H|$

$$\prod_{G/H} \mathcal{C} \longrightarrow \mathcal{C}$$

$$C_1, \dots, C_j \longmapsto C_1 \oplus C_2 \oplus \dots \oplus C_j$$

not equivariant since  
 $G$  permutes on the left but  
 not on the right.

Definition Recall 0th component of KR is

$$\coprod_n BGL_n(\mathbb{R})$$

Idea: replace these by equiv bundles.

Thm (Quillen - May - M)  $G$  finite,  $\Pi$  compact Lie

$\tilde{C}_G =$  translation category

$B\tilde{C}_G \cong EG$

$\text{Cat}(\mathcal{P}, \mathcal{D}) =$  all functors  
+ nat trans  
of  $G$ -categories

$B\text{Cat}(\tilde{C}_G, \Pi) \rightarrow B\text{Cat}(G, \Pi)$  is  
a universal  $\Pi \times G$ -bundle

Guess  $K_G(\mathbb{R}) =$  gp completion of  
 $\coprod B\text{Cat}(\tilde{C}_G, GL_n(\mathbb{R}))$   
Is this an  $S^{\infty} G$ -space?

Def  $\text{Cat}(\tilde{G}, \rho)_{G_0} = \mathcal{C}^{hG}$   
 = lity fixed points  
 of  $\mathcal{C}$

$E_{\text{top}}$ -operads  
 Top

	$E_{\text{top}}$	$\text{Cat}$
non equiv	$\mathcal{O}(j) = E\Sigma_j$	$B\mathcal{O}(j) = E\Sigma_j \sim \Sigma_j$ $\mathcal{O}(j) = \Sigma_j$
equiv	$\mathcal{O}_G = \text{universal } G \times \Sigma_j \text{-bundle}$	$\mathcal{O}_G(j) = \text{Cat}(\tilde{G}, \tilde{\Sigma}_j)$ (Dunkl-May)

FACT (May):  $\text{permutative cats} \cong \text{alg} / \mathcal{O}$  strict SMC  
symm monoidal cats  $\cong \text{pseudo alg} / \mathcal{O}$ .

Def (Quillen-May) A genuine perm  $n$ -cat is an algebra  $/ \mathcal{O}_n$ .

A genuine SMC is a pseudo-algebra  $/ \mathcal{O}_n$ .