MATH 412: TOPICS IN ALGEBRA
FINAL PROJECT

In your final project in Math 412, you will explore and explain a special topic in quadratic forms through a 6–10 page paper and a 20-minute presentation.

Topic selection and proposal. You are welcome to propose any topic related to quadratic forms which is distinct from those covered in the course. (After break, we are going to cover Pfister forms, quaternion algebras, and Hilbert reciprocity.) If you are looking for an idea, you might want to investigate the following:

- Integral quadratic forms — binary forms following Conway, or the 15 and 290 theorems of Bhargava–Hanke.
- Milnor K-theory — the Milnor conjecture, Hilbert reciprocity via symbols, Hasse–Witt and Stiefel–Whitney invariants.
- The Witt ring of $\mathbb{Q}$.
- Classification of small Witt rings.
- The prime ideals of the Witt ring.
- Trace forms — computations and properties for a Galois extension, Mackey functor structure of the Grothendieck–Witt ring, relation to the discriminant of a number field.
- Quadratic forms as the degree of a rational function (see Cazanave, *Algebraic homotopy classes of rational functions*).
- Quadratic enumerative geometry (following Kass–Wickelgren).
- Alternating and Hermitian forms.
- Symmetric bilinear and quadratic forms in characteristic 2.
- The Hasse principle.
- Applications of quadratic forms in other fields of mathematics.

I am happy to consult on topics and help you track down appropriate references.

You must propose first and second choice topics by 1 November. Please send an email to me listing your preferences, along with one or two setences describing what you would cover and one or two resources you might draw from. Topics will be assigned first-come-first-serve to avoid duplication.

Final paper. You will write a 6–10 page paper on your selected topic. You must write the paper using LaTeX in the `amsart` document class using the page layout specified in the template file `http://people.reed.edu/~ormsbyk/412/paper_template.tex`. Resources for learning LaTeX are available on the course website.

You must submit a draft (at least four pages long) at the start of class on 22 November. You will receive comments on your draft by 25 November. After incorporating these comments, you will **submit your final paper at the start of class on 2 December.**

The audience for your paper is a contemporaneous Math 412 student. Your paper should create interest in the topic you are exploring, explain its context, and present some of the pertinent results with proofs. You should assume the material presented in class, but nothing beyond. Please use LaTeX's sectioning, numbering, and environment features to format your paper in typical mathematical style; please use BibTeX for your bibliography.

Final presentation. You will also give a 20-minute presentation on your topic in class on an assigned date between 2 December and 11 December. You will schedule a practice talk with me before your talk and are encouraged to incorporate feedback from that meeting into your talk. The talk may be chalk- or slide-based. (You can do LaTeX slides via the `beamer` package, but there are other options as well.) Given the time constraints, your talk should focus on concepts and theorem statements, but you should also sketch at least one proof. Finally, you will assign one homework problem to the class based on your topic.

You will also act as the audience for your classmates' presentations. In this role, you will provide written feedback to presenters and complete their homework problems.

Learning goals and assessment. As you progress as a mathematician, you will need to independently learn and communicate material, in both written and verbal forms. Your final project will provide a structure in which to practice these skills, including LaTeX document preparation and presentation skills pertinent to your Reed thesis work. By submitting a draft paper and giving a practice presentation, you will have the opportunity to learn from initial mistakes and improve your material.

These papers and presentations are also an opportunity for the class to share its interests with each other and for all of us to learn about additional topics in algebra. For some of you, it will be the first of many opportunities to teach mathematics to others. You are expected to participate as an active audience member, to provide feedback on your peers' presentations, and to complete/attempt their homework problems.

Your final paper will be assessed based on the following characteristics (in roughly descending order of importance):

- Mathematical content and precision.
- Mathematical context and narrative.
- Style, including clarity and grammar.
- LaTeX formatting.

Your presentation will be assessed based on the following characteristics (again in roughly descending order of importance):

- Clarity of ideas and information.
- Organization and narrative.
- Clarity of board work or slides.
- Speaking volume and body language.
- Response to audience questions.
- Appropriateness of homework problem.

You will also receive holistic comments on the overall quality of your paper and presentation.