

MATH 412: TOPICS IN ALGEBRA
HOMEWORK DUE FRIDAY WEEK 11

Problem 1. Given $A = \left(\frac{a,b}{k}\right)$, consider the following statements:

- (a) -1 is a square in A .
- (b) $A \cong \left(\frac{-1,c}{k}\right)$ for some $c \in k^\times$.
- (c) $\langle 1, a, b, -ab \rangle$ is isotropic.
- (d) There exists $z \in A \setminus \{0\}$ such that z^2 is a pure quaternion.

Show that (d) $\iff$ (c) $\iff$ (b) $\implies$ (a), and that (a) $\implies$ (b) if $-1 \notin k^{\square}$.

Problem 2. Use condition (g) of Theorem 21.19 in the course notes to show that $A = \left(\frac{3,-11}{\mathbb{Q}}\right)$ splits. (Feel free to reference facts from number theory if needed.) Find a nonzero element $z \in A$ such that $z^2 = 0$.