MATH 412: TOPICS IN ALGEBRA

FALL 2019

Place:	Eliot 207
Time:	MWF, 2:40–3:30P.M.
Instructor:	Kyle Ormsby (ormsbyk@reed.edu)
Office Hours:	Library 306, M 3:30–4:30P.M., Th 2:30–4:00P.M.
Textbook:	None required; see recommendations below
Website:	people.reed.edu/~ormsbyk/412/

Summary. This iteration of Math 412 will investigate the theory of quadratic and symmetric bilinear forms. A quadratic form is a homogeneous polynomial of degree 2 in several variables. They arise throughout mathematics, including the second partial derivative test in multivariable optimization, sums of squares formulas in number theory, lattice packings of spheres, and enumerative algebraic geometry. Our study will focus on the following topics:

» prerequisites from linear algebra (tensor products, duals),
» faces of quadratic forms (polynomials, symmetric bilinear forms, quadratic spaces, symmetric matrices),
» diagonalization and Witt cancellation,
» the Witt and Grothendieck–Witt rings,
» ordered fields and signature theory,
» Pfister forms and sums of squares, and
» quaternion algebras and Hilbert reciprocity.

Time permitting, we will also look at the integral theory of quadratic forms and some applications to other branches of mathematics.

At the end of the course, students should feel comfortable with the algebraic theory of quadratic forms, understand uses and computations of Grothendieck–Witt rings for various fields, and be able to relate properties of fields to properties of quadratic forms.

Texts. The course has no assigned textbook. The instructor will write course notes as the semester proceeds and post these to the course website. Supplementary texts which may prove useful include *Introduction to quadratic forms over fields* by Lam, *Symmetric bilinear forms* by Milnor and Husemoller, and *Bilinear algebra* by Szymiczek.

Homework. Homework is due most Fridays in the bin outside my office, Library 306, by 4:00P.M. Excellent solutions take many forms, but they all have the following characteristics:

» they are written as explanations for other students in the course; in particular, they fully explain all of their reasoning and do not assume that the reader will fill in details;
» when graphical reasoning is called for, they include large, carefully drawn and labelled diagrams;
» they are neatly written or typeset;[1] and

[1] Interested students are encouraged to prepare solutions in the LaTeX document preparation system. A guide to LaTeX resources is available on the course website. Nearly all of the .pdf files on the course website are produced by LaTeX; you can find their associated source files by changing the .pdf suffix to .tex in the URL.

» they use complete sentences, even when formulas or symbols are involved.

Because of time constraints, I will mark some homework problems for completion and others (semi-randomly selected) fully. Fully graded homework problems can earn up to five points for mathematical content and will also have the quality of writing assessed with a ✓+, ✓, or ✓−. I reserve the right to not accept late homework. If health or family matters might impede the timely completion of your homework, please contact me as early as possible.

Collaboration. You are permitted and encouraged to work with your peers on homework problems. You must cite those with whom you worked, and you must write up solutions independently. **Duplicated solutions will not be accepted and constitute a violation of the Honor Principle.**

Revisions. You may revise any submitted fully graded homework problem after receiving comments, and you will sometimes be encouraged to revise problems. (You may not "revise" homework problems that were not turned in the first time.) This will allow you the opportunity to perfect the skills required to solve the problems. You may revise multiple times, and will receive the average of all of your scores. Revisions must be turned in at most one week after you receive comments on the previous version of a solution.

Tests. We will have two timed take-home exams. You may reference one two-sided US Letter or A4-size page of notes during each exam. Calculators, computers, phones, collaboration, books, and the Internet are prohibited during exams.

» Exam 1: two hours, distributed Monday, 14 October, due Friday, 18 October.
» Exam 2: three hours, distributed during finals week.

Presentations and final paper. Each student will give a 20-minute presentation on an advanced topic in the final two weeks of the course. You will also submit a final paper on the same topic. More details will follow, but expect that topics will be assigned early, that you will give a practice presentation, and that comments will be given on draft papers (prepared in LaTeX) before the final version is submitted.

Joint expectations. As members of a communal learning environment, we should all expect consideration, fairness, patience, and curiosity from each other. Our aim is to all learn through cooperation and genuine listening and sharing, not to compete or show off. I expect diligence and academic and intellectual honesty from each of you. You should expect that I will do my best to focus the course on interesting, pertinent topics, and that I will provide feedback and guidance which will help you excel as a student.

Help. Everyone is welcome and encouraged to attend my office hours, Monday 2:30–4:30P.M. and Thursday 2:30–4:00P.M. in Library 306. If you are unable to make these times, I am happy to schedule alternate times at which to meet with you — just ask!

Piazza. Our section of 412 has a Piazza page on which you can ask and answer questions. Sign up at this link. This course will attempt to conduct all electronic conversations through Piazza. This has the benefit of making questions and the ensuing conversations visible to the entire class. And don't worry — you can post anonymously if you prefer. The Piazza page is an extension of our classroom and Reed, and the Honor Principle and our joint expectations govern our conversations there.

Technology. The use of electronic devices (cell phones, computers, tablets, calculators, *&c*) is strictly prohibited in the classroom without prior authorization from the instructor. That said, legitimate uses of technology (*e.g.*, note-taking) will be accommodated — just talk to me first.

The Internet. You are welcome to use Internet resources to supplement content we cover in this course, with the exception of solutions to homework problems. **Copying solutions from the Internet is an Honor Principle violation and will result in an academic misconduct report.**

Academic accommodations. If you have a documented disability requiring academic accommodation, please have Disability Support Services (DSS) provide a letter during the first week of classes. I will then contact you to schedule a meeting during which we can discuss your accommodations. If you believe you have an undocumented disability and that accommodations would ensure equal access to your Reed education, I would be happy to help you contact DSS.

Grades. Your grade will reflect a composite assessment of the work you produce for the class, weighted in the following fashion: 25% homework, 20% exam 1, 20% exam 2, 20% final paper, 10% final presentation, 5% class participation.

Remember: *Math is hard, but we're going to get through this together!*