MATH 342: TOPOLOGY
HOMEWORK DUE FRIDAY WEEK 11

Problem 1. Let X be a topological space and $f\colon X \to S$ a surjective function. Define an equivalence relation on X by $x \sim x' \iff f(x) = f(x')$. Let

$$R = \{(x, x') \in X \times X \mid f(x) = f(x')\}.$$

Let r_1 and r_2 be the composition of the inclusion $R \hookrightarrow X \times X$ with the projection maps π_1 and π_2, respectively. Prove that S with the quotient topology is the coequalizer of $r_1, r_2\colon R \rightrightarrows X$.

Problem 2. Let $f\colon Y \to X$ be an embedding of a space Y into a space X. Construct a diagram for which Y (along with the map f) is a limit. (*Hint*: Use something similar in flavor to what you built in the previous problem.)

Problem 3. For any set X, show that the functor $X \times -\colon \mathsf{Set} \to \mathsf{Set}$ preserves colimits, *i.e.*, is cocontinuous. (You need to specify what $X \times -$ means as a functor and show that $\operatorname{colim}(X \times F) = X \times \operatorname{colim} F$ for all diagrams F in Set).

Problem 4. Suppose that $L\colon \mathsf{C} \leftrightarrows \mathsf{D} : R$ is a pair of adjoint functors with unit η and counit ε. Prove that η and ε satisfy the *triangle identities* given in Definition 5.2 of the text. (Optional: Also show that Definition 5.2 implies Definition 5.1, so that both definitions of adjoint pairs are equivalent.)