MATH 342: TOPOLOGY
HOMEWORK DUE FRIDAY WEEK 7

Problem 1. Prove that the (arbitrary) product of Hausdorff spaces is Hausdorff.

Problem 2. Let X be a space and suppose $A, B \subseteq X$ are compact subspaces. Prove or disprove:
(a) $A \cap B$ is compact.
(b) $A \cup B$ is compact.

Problem 3. Recall that ℓ_2 is the space of square summable sequences with metric

$$d(x, y) = \sqrt{\sum (x_i - y_i)^2}.$$

Prove that $D(0, 1) = \{(x_i) \mid \sum x_i^2 \leq 1\} \subseteq \ell_2$ is not compact.

Problem 4. For any function $f\colon X \to Y$, the set $\Gamma_f = \{(x, f(x)) \mid x \in X\} \subseteq X \times Y$ is called the *graph* of f. Suppose X is any space and Y is a compact Hausdorff space. Prove that Γ_f is closed if and only if f is continuous.

Problem 5. Consider $\mathbb{Q}$ with its standard metric. Find a closed and bounded subset of $\mathbb{Q}$ which is *not* compact.