MATH 342: TOPOLOGY
HOMEWORK DUE FRIDAY WEEK 6

Problem 1. A map $f\colon X \to Y$ is *locally constant* when for each $x \in X$ there exists an open neighborhoood U of x such that $f|_U$ is constant. Prove or disprove: if X is connected and Y is any space, then every locally constant map $f\colon X \to Y$ is constant.

Problem 2. Recall that $\mathbb{R}^{\mathbb{N}}$ is the space sequences in $\mathbb{R}$ endowed with the product topology. There is another topology on $\mathbb{R}^{\mathbb{N}}$ called the *box topology* that has basis of open sets

$$\mathscr{B}_{\text{box}} = \left\{ \prod_{n \in \mathbb{N}} U_n \;\middle|\; U_n \subseteq \mathbb{R} \text{ open for all } n \in \mathbb{N} \right\}.$$

We denote this space $\mathbb{R}^{\mathbb{N}}_{\text{box}}$. Prove that $\mathbb{R}^{\mathbb{N}}$ is connected while $\mathbb{R}^{\mathbb{N}}_{\text{box}}$ is not connected.

Problem 3. Prove that there are two antipodal points on the Earth's equator that have the same temperature.

Problem 4. Show that if A is a countable subset of $\mathbb{R}^2$, then $\mathbb{R}^2 \setminus A$ is path connected. (*Hint*: How many lines are there passing through a given point of $\mathbb{R}^2$?)