MATH 341: TOPICS IN GEOMETRY
HOMEWORK DUE FRIDAY WEEK 11

Problem 1. Let S denote the sum of the interior angles of a planar hyperbolic polygon P with n sides. Prove that

$$\text{area}(P) = (n-2)\pi - S.$$

(In particular, we always have $S < (n-2)\pi$.)

Problem 2. Let D be a hyperbolic disk of radius r centered at i in the upper half-plane H^2. We know that D is a Euclidean disk as well.

(a) Prove that the Euclidean center of D is $i(e^r + e^{-r})/2 = i\cosh(r)$ and that its Euclidean radius is $(e^r - e^{-r})/2 = \sinh(r)$.

(b) Use polar coordinates centered at the Euclidean center you found in (a) to compute

$$\text{area}(D) = \int_D \frac{1}{y^2}.$$

Since isometries preserve area and any hyperbolic disk of radius r is isometric to D, you have just computed the area of a hyperbolic disk in general.

Problem 3. The group $(\mathbb{R}, +)$ acts on $\mathbb{R}/2\pi\mathbb{Z}$ via $x \cdot (y + 2\pi\mathbb{Z}) = (x + y) + 2\pi\mathbb{Z}$. Show that the $\mathbb{Z}$-orbit of $1 + 2\pi\mathbb{Z}$ is not discrete.