MATH 341: TOPICS IN GEOMETRY
HOMEWORK DUE FRIDAY WEEK 10

Problem 1. Let $ABCD$ be a *parallelogram* in $\mathfrak{h}$, that is, a convex quadrilateral in which opposite sides have equal length.

(a) Show that the diagonals AC and BD bisect each other.

(b) Are the opposite sides of a hyperbolic parallelogram necessarily parallel?

Hint: Show that the half-turn μ with respect to the midpoint M of AC takes the quadrilateral to itself.

Problem 2. Read the definition of a *horocycle* with center $S \in \partial\mathfrak{h}$ on the bottom of p.100 of Iversen, and then read Corollary 3.4 and its proof.

(a) Draw a family of horocycles with center 0 in the Poincaré upper half plane.

(b) Fix $S \in \partial\mathfrak{h}$. Show that the horocycles with center S are of the form $P \cap \mathfrak{h}$ where P is an affine plane in $\mathfrak{sl}_2(\mathbb{R})$ parallel to $T_S(\mathfrak{h}) = S^\perp$.

Problem 3. Let $A \in \mathfrak{sl}_2(\mathbb{R})$ represent a point of $\mathfrak{h}$. Show that the transformation $X \mapsto AXA^{-1}$ of $\mathfrak{sl}_2(\mathbb{R})$ is a half-turn with respect to the point A.