MATH 341: TOPICS IN GEOMETRY
HOMEWORK DUE FRIDAY WEEK 5

Problem 1. Let $\mathbb{H}^1$ denote the upper sheet of $S(\mathbb{R}^{-1,1})$. Set $c = (\sqrt{3}, \sqrt{2}) \in \mathbb{R}^{-1,1}$ and note that it has norm -1. Explicitly compute τ_c, reflection along c in $\mathbb{R}^{-1,1}$, and draw a picture exhibiting how it acts on $\mathbb{H}^1$.

Problem 2. Let D^2 denote the 2-dimensional Klein disk. Draw a family of circles along a diameter of D^2 where the radius of each circle is 1 (measured via the Klein hyperbolic metric). Justify your calculations and picture. Conclude by observing that while lines are easy to visualize in D^2, circles are less pleasant.

Problem 3. Verify the formulæ for $p\colon D^n \to \mathbb{H}^n$ and $f\colon D^n \to \mathbb{H}^n$ given in the notes. (These maps are defined in terms of intersections between certain lines and a particular model of $\mathbb{H}^n$. You need to check that the intersection points are in fact given by the formulæ in the notes.)

Problem 4. Prove Ptolemy's theorem:

> Let E be a Euclidean vector space of dimension n. Then $n+2$ points $x_1, \ldots, x_{n+2} \in E$ lie on a sphere if and only if $\det(d(x_i, x_j)^2)_{i,j} = 0$, where d denotes Euclidean distance.

Hint: Show that $\langle \iota x, \iota y \rangle = \frac{1}{2} d(x,y)^2$ for $x, y \in E$.