

MATH 341: TOPICS IN GEOMETRY

SPRING 2020

Place:	Chem 301
Time:	MWF, 1:10–2P.M.
Instructor:	Kyle Ormsby (ormsbyk@reed.edu)
Office Hours:	Tu, Th 2–3P.M.
Textbook:	<i>Hyperbolic geometry</i> by Birger Iversen
Website:	people.reed.edu/~ormsbyk/341/

Summary. This iteration of Math 341 will investigate hyperbolic geometry, a non-Euclidean geometry that does not satisfy the parallel postulate:

Given a line and a point not on it, there is exactly one line going through the given point that is parallel to the given line.

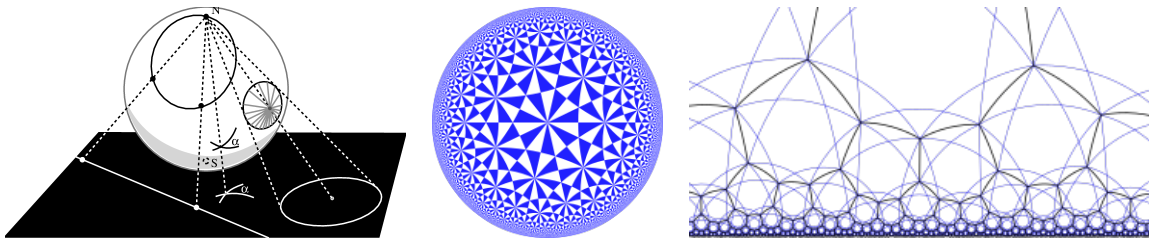
Despite having arisen in the first half of the nineteenth century from purely mathematical investigations into the basis of axiomatic geometry, hyperbolic geometry has proven essential throughout various fields of applied and pure mathematics. It underpins Einstein's notion of spacetime, plays a crucial role in complex analysis, and spawned the field of geometric group theory.

This course will naturally split into two main themes:

- (1) models of hyperbolic space, and
- (2) Fuchsian groups and Poincaré's Theorem.

After developing some preliminary material on quadratic forms and orthogonal groups, the first portion will introduce the five canonical models of hyperbolic space (in arbitrary dimension) and investigate the properties of geodesics, polygons, &c. in these settings. The second portion will specialize to the hyperbolic plane and its discrete subgroups of isometries, *i.e.*, the *Fuchsian groups*. The course will culminate in Poincaré's Theorem, which identifies the hyperbolic polygons (with side pairings) that generate Fuchsian groups.

This subject is intensely visual, and special attention will be paid to the presentation of appropriate illustrations:



Learning objectives. At the end of the semester, students should understand the two main themes of the course and exhibit proficiency in solving related problems. They will also gain skill in writing and presenting mathematical arguments, and develop the mathematical maturity necessary to engage with advanced material both in a class setting and independently.

Texts. The required textbook for the course is *Hyperbolic geometry* by Birger Iversen. Additional resources will be posted on the course website.

Homework. Homework is due most Fridays at the start of class. Excellent solutions take many forms, but they all have the following characteristics:

- » they are written as explanations for other students in the course; in particular, they fully explain all of their reasoning and do not assume that the reader will fill in details;
- » when graphical reasoning is called for, they include large, carefully drawn and labelled diagrams;
- » they are neatly written or typeset;¹ and
- » they use complete sentences, even when formulas or symbols are involved.

Because of time constraints, I will mark some homework problems for completion and others (semi-randomly selected) fully. Fully graded homework problems can earn up to five points for mathematical content and will also have the quality of writing assessed with a $\checkmark+$, \checkmark , or $\checkmark-$. I reserve the right to not accept late homework. If health or family matters might impede the timely completion of your homework, please contact me as early as possible.

Collaboration. You are permitted and encouraged to work with your peers on homework problems. You must cite those with whom you worked, and you must write up solutions independently. **Duplicated solutions will not be accepted and constitute a violation of the Honor Principle.**

Tests. We will have three timed take-home exams. You may reference one two-sided US Letter or A4-size page of notes during each exam. Calculators, computers, phones, collaboration, books, and the Internet are prohibited during exams.

- » Exam 1: two hours, distributed Monday, 24 February, due Friday, 28 February.
- » Exam 2: two hours, distributed Monday, 30 March, due Friday 3 April.
- » Exam 3: three hours, distributed Monday 27 April, due Friday 1 May.

Final presentations. Students will give a 20-minute slide presentation in teams of two during the final exam time allotted by the registrar. **Make certain that you do not schedule any travel that conflicts with the exam time (which will be announced sometime in mid-February).** This project will give you the chance to explore an advanced topic in hyperbolic geometry with a peer and present your findings with the visual aid of slides.

Joint expectations. As members of a communal learning environment, we should all expect consideration, fairness, patience, and curiosity from each other. Our aim is to all learn through cooperation and genuine listening and sharing, not to compete or show off. I expect diligence and academic and intellectual honesty from each of you. You should expect that I will do my best to focus the course on interesting, pertinent topics, and that I will provide feedback and guidance which will help you excel as a student.

Help. Everyone is welcome and encouraged to attend my office hours, Monday and Thursday 2–3P.M. in Library 306. If you are unable to make these times, I am happy to schedule alternate times. One convenient way to schedule these times is via the following link: <https://calendly.com/kyleormsby/math-meeting>.

¹Interested students are encouraged to prepare solutions in the \LaTeX document preparation system. A guide to \LaTeX resources is available on the course website. Nearly all of the .pdf files on the course website are produced by \LaTeX ; you can find their associated source files by changing the .pdf suffix to .tex in the URL.

Moodle forum. While the primary website for this course is the one listed at the top of the first page, we will also have a Moodle site hosting a forum. You can use the forum to ask questions of me or the class, and I am hopeful that we can use this resource to productively share hints, insights, and additional resources. The forum is an extension of our classroom and the above joint expectations extend to this setting.

Course announcements. All course announcements will be made in class or via email.

Technology. The use of electronic devices (cell phones, computers, tablets, calculators, &c) is strictly prohibited in the classroom without prior authorization from the instructor. That said, legitimate uses of technology (*e.g.*, note-taking) will be accommodated — just talk to me first.

The Internet. You are welcome to use Internet resources to supplement content we cover in this course, with the exception of solutions to homework problems. **Copying solutions from the Internet is an Honor Principle violation and will result in an academic misconduct report.**

Academic accommodations. If you have a documented disability requiring academic accommodation, please have Disability & Accessibility Resources (DAR) provide a letter during the first week of classes. I will then contact you to schedule a meeting during which we can discuss your accommodations. If you believe you have an undocumented disability and that accommodations would ensure equal access to your Reed education, I would be happy to help you contact DAR.

Grades. Your grade will reflect a composite assessment of the work you produce for the class, weighted in the following fashion: 25% homework, 15% exam 1, 15% exam 2, 20% exam 3, 20% final presentation, 5% class participation.

Remember: *Math is hard, but we're going to get through this together!*