Lecture 49

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Linear map
$$T: V \longrightarrow V$$
, $V = fin dim F \cdot vs$.
Suppose $c_T(x)$ has all its roots in F .
Then $c_T(x) = TT((x-\lambda_i)^k$:
 $k_i \in \mathbb{Z}^+$, $\lambda_i \in F$

This all elementary divisors of
$$F[x]$$
-module V_7
are of the form $(x - \lambda)^{le}$
Study $F[x]/((x-\lambda)^{le})$:

$$(\bar{x}-\lambda)^{k-l}$$
, $(\bar{x}-\lambda)^{k-2}$, ..., $\bar{x}-\lambda$,

 $(\overline{z}-\lambda)^{j} = \sum_{i=0}^{j} {\binom{j}{i} (-\lambda)^{j-i} \overline{z}^{i}}$

$$= (-\lambda)^{j} + j (-\lambda)^{j-1} + {\binom{j}{z}} (-\lambda)^{j-2} \overline{x}^{2} + \cdots + \overline{x}^{j}$$

so the matrix expressing $(\overline{x} - \lambda)^{j} \cdot j$ in terms of $(1, \overline{z}, \overline{x}, \dots, \overline{x}^{k})$
 $= \frac{1}{x} \cdot \frac{1}{x}$

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is of the form
$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ * & 1 \end{pmatrix}$$
 where 1

$$\implies (\bar{x}-\lambda)^{k-1}, (\bar{x}-\lambda)^{k-1}, \dots, 1 \quad \text{is a basis} \; !$$

How does x act on this basis? $\lambda + (x - \lambda)$

$$(\bar{x} - \lambda)^{k-1} \xrightarrow{x} \lambda (\bar{x} - \lambda)^{k-1} + (\bar{x} - \lambda)^{k} = \lambda (\bar{x} - \lambda)^{k-1}$$

$$(\bar{x} - \lambda)^{k-2} \xrightarrow{x} \lambda (\bar{x} - \lambda)^{k-2} + (\bar{x} - \lambda)^{k-1}$$

$$(\bar{x}-\lambda)^{j} \xrightarrow{x} \lambda (\bar{x}-\lambda)^{j} + (\bar{x}-\lambda)^{j+1}$$
The matrix for x on $F[x]/((x-\lambda)^{k})$ with chosen basis is

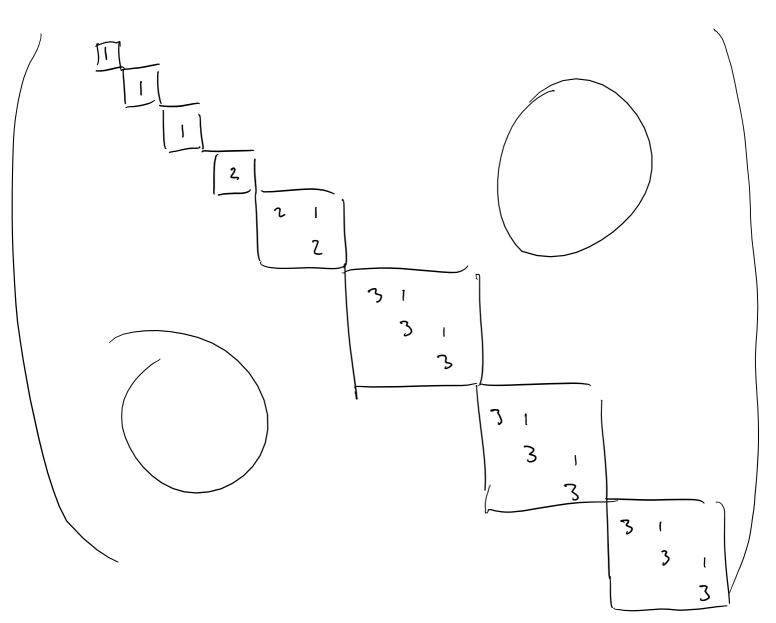
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Elementary divisor decomposition of V7: $V_{T} \cong \bigoplus_{i=1}^{t} \frac{F(x)}{\left((x-\lambda_{i})^{k_{i}}\right)}$

Can choose a basis s.t. the matrix for T is of $\left(\begin{array}{ccc} J_{i} & 0 \\ J_{2} & 0 \\ 0 \\ J_{t} \end{array}\right) \begin{array}{c} \mbox{where} & J_{i} & if the \\ \mbox{J}_{t} & J_{t} \\ \lambda_{i} \end{array}$ th firm This the Jordan canonical form of T, Then If the eigenvalues of Taxe all in F, then F basis of Vr.t. T is in JCF withing basis and this form is unique up to permutation of the Jordan élochs. \Box

e.g. Suppose we know the invariant factors of T

$$(x-1)(x-3)^{3}$$
, $(x-1)(x-2)(x-3)^{3}$, $(x-1)(x-2)^{2}(x-3)^{3}$
The elementary divisors of T are
 $x-1$, $(x-3)^{3}$, $x-1$, $x-2$, $(x-3)^{3}$, $x-1$, $(x-2)^{2}$, $(x-3)^{3}$



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$$Q I_{5} A = \begin{pmatrix} 2 & -2 & 14 \\ 0 & 3 & -7 \\ 0 & 0 & 2 \end{pmatrix}$$
 diagonalizable ?

$$\underline{A} = \mathbf{c}_{\mathbf{A}}(\mathbf{x}) = dat(\mathbf{x}\mathbf{I}-\mathbf{A})$$