Review

Friday, March 6, 2015 9:56 AM

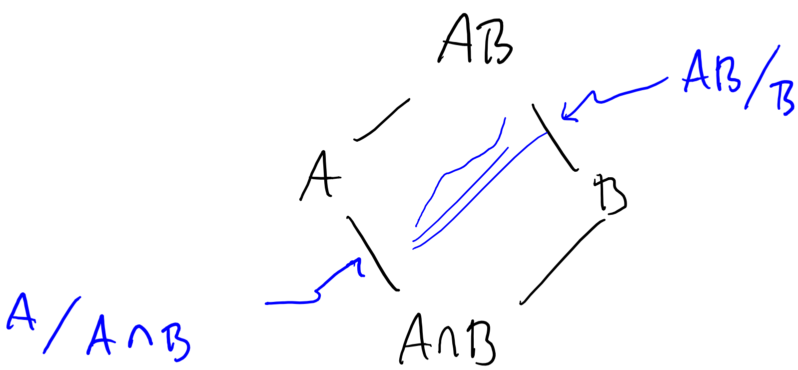
Iso Thms

① $G \xrightarrow{\varphi} H$ hom then $\ker(\varphi) \trianglelefteq G$

and $\operatorname{im}(\varphi) \cong G/\ker(\varphi)$.

② $A, B \leq G$, $A \leq N_G(B)$. Then $AB \leq G$,

$B \trianglelefteq AB$, $A \cap B \trianglelefteq A$, and $AB/B \cong A/A\cap B$



③ $H, K \trianglelefteq G$, $H \leq K$. Then $K/H \trianglelefteq G/H$ &

$(G/H)/(K/H) \cong G/K$.

④ $N \trianglelefteq G$. Then $\{A \leq G \mid A \geq N\} \xleftrightarrow{\text{bij}} \{\bar{A} \leq G/N\}$.

$A \longmapsto A/N = \mathrm{red}(A)$

$G \xrightarrow{\mathrm{red}} G/N$

$G \xrightarrow{red} G/N$

$A \longmapsto A/N = red(A)$

$red^{-1}(\bar{A}) \longleftarrow\!\!\shortmid \bar{A}$

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Universal property

$$\begin{tikzcd} H \arrow[r, hook, "\trianglelefteq"] \arrow[dr, "1"'] & G \arrow[r, "\text{red}"] \arrow[d, "\varphi"] & G/H \arrow[dl, dashed, "\exists!\,\bar\varphi"] \\ & K & \end{tikzcd}$$

$g \mapsto gH = ghH$ for any $h \in H$

$gh \mapsto ghH$

$\varphi(g) = \varphi(gh)$

$\cancel{\varphi(g)} = \varphi(gh)$

$= \cancel{\varphi(g)}\varphi(h)$

$1 = \varphi(h)$

If $\varphi : G \longrightarrow K$ is a hom and $H \trianglelefteq G$, then $\exists\ \bar\varphi : G/H \longrightarrow K$ a hom s.t. $\varphi = \bar\varphi \circ \text{red}$ iff $\varphi(H) = 1$ (i.e. $H \leq \ker(\varphi)$). When such a $\bar\varphi$ exists, it is unique.

Comment Any $G \xrightarrow{\psi} Q$ w/ $\ker(\psi) = H$ satisfying the above property is isomorphic to G/H.

Jordan-Hölder program: any finite gp G has a composition series

$$1 = G_0 \trianglelefteq G_1 \trianglelefteq G_2 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$$

w/ G_{i+1}/G_i simple

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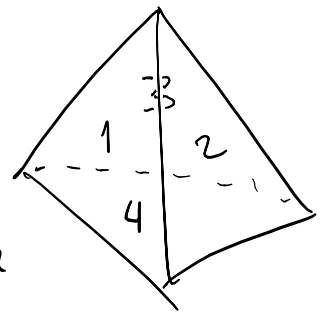
Orbit - Stabilizer



T = rigid symms of

$|T| = ?$ $T \circlearrowright \{\text{faces}\} = \{1,2,3,4\} = \underline{4}$

For $i \in \underline{4}$, OST says $|T \cdot i| = |T| / |T_i|$

$$\Longrightarrow |T| = |T \cdot i| \cdot |T_i|$$

Take $i = 4$. $|T \cdot 4| = 4$

$$|T_4| = 3$$

Thus $|T| = 12$.

Claim $T \cong A_4$.

Pf $T \circlearrowright \{\text{faces}\}$ gives permutation rep

$T \xrightarrow{\varphi} S_4$. Since the only symm of T leaving all faces fixed is id, $T \xrightarrow[\varphi]{} S_4$ has trivial kernel & thus is injective.

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$\forall \sigma \in T$. $\varphi(\sigma) = \begin{cases} id \\ 3\text{-cycle} \end{cases}$ and thus

the image of φ consists of even permutations,

i.e., $\mathrm{im}(\varphi) \subseteq A_4$ and both sets have size 12

$\Rightarrow \mathrm{im}\,\varphi = A_4$. $\square$

T generated by 3-cycles.

$$\underbrace{(1\ 2)(3\ 4)}_{\substack{i.2 \\ T}} \in A_4$$

$$\| $$

$$(1\ 3\ 2)(1\ 3\ 4)$$

$G \underset{\text{conj}}{\circlearrowright} G$ + OST $\leadsto$ Class eq'n.

For $|G| < \infty$, $|G| = |Z(G)| + \sum_{x \in A} [G : \underbrace{C_G(x)}]$

A is a set of reps of conj classes in G of size > 1.

stabilizer of x under conj action.