MATH 138: WEEK 9 HOMEWORK

Exercise 1 (Adams Exercise 6.14). Compute the Alexander polynomial of the figure-eight knot by hand. Use `SnapPy` to verify your answer.

Exercise 2 (Adams Exercise 6.25). Suppose that K is an alternating knot with odd crossing number. Prove that K is *not* amphichiral.

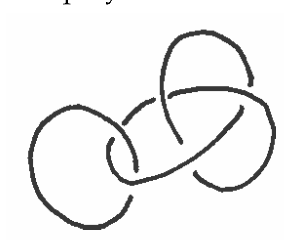
Exercise 3. For an arbitrary link L, the Jones polynomial $V(L)$ is a polynomial in $t^{1/2}$.

(a) For a positive integer n, let K_n denote the n component split unlink. Prove that $V(K_n) = t^{(n-1)/2}(-t^{-1} - 1)^{n-1}$.
(b) Use the skein relation to argue that if any two of $V(L_+)$, $V(L_-)$, and $V(L_0)$ are polynomials in t, then the third is a polynomial in t as well. (Note: Being a polynomial in t is a stronger condition than being a polynomial in $t^{1/2}$.)
(c) Use part (b) and an analysis of the parities of the number of components of L_+, L_-, and L_0 in order to argue that any link L with an odd number of components has $V(L)$ a polynomial in t.

Exercise 4. Prove that for any link L, $V(L)(-1) = \Delta(L)(-1)$, *i.e.*, that the evaluations of the Jones and Alexander polynomials at -1 are equal. This number is called the *determinant* of L, $\det(L)$.

Exercise 5. For any link L, prove that $\det(L) = \det(L^*)$, where L^* is the mirror image of L.

Exercise 6. Compute the Jones polynomial of



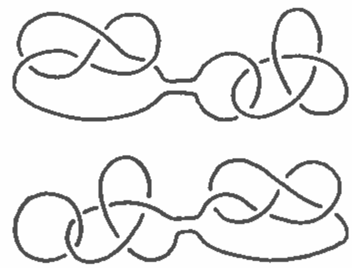
by hand. Use `SnapPy` to verify your answer.

Exercise 7. Use your answer to the previous exercise, known computations of the Jones polynomial of the trefoil and figure eight knots, and the fact

$$V(L_1 \# L_2) = V(L_1) \cdot V(L_2)$$

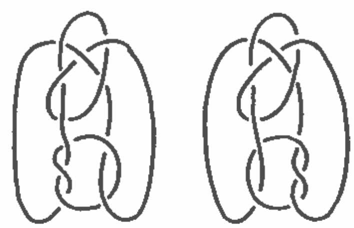
to prove that the following two links have identical Jones polynomials.

Date: 27.IV.15.



Bonus: Prove that the two links are nonisotopic.

Exercise 8. Use `SnapPy` to check that the following two knots have the same Jones polynomial.



Bonus: Prove that two knots are nonisotopic.

Exercise 9 (*Bonus*). If `L` is a link in `SnapPy`, then `L.determinant()` determines the determinant of `L`. Find the first nontrivial knot with determinant ± 1. (The answer is pretty far out in the Rolfsen knot table. I *highly* recommend learning enough `sage` to automatically iterate through the table.)