MATH 113: DISCRETE STRUCTURES
FINAL EXAM REVIEW

GP = group problems avaible on Dave Perkinson's Math 113 site, http://people.reed.edu/~davidp/113/.
Also see the slide presentations on Dave's site for additional excellent study material.

» Basic counting
  – Additive and multiplicative counting principles
  – Factorials, binomial coefficients, Pascal's triangle
  – Statement of the binomial theorem
  – *Problems:* HW3W, HW4M, HW4F, HW5M, HW6W Problem 1, HW7F Problem 1, GP3F Problems 2 and 3, GP4W, Exam 1 Q1, Q2, Q3, Exam 2 Q1
» Equivalence relations
  – *Problems:* HW3F, Exam 1 Q4, Q5
» Induction proofs
  – *Problems:* HW5W Problem 1, GPF4 Problem 1
» Inclusion/exclusion
  – *Problems:* HW5F Problem 2, Exam 2 Q3
» Pigeon-hole principle
  – *Problems:* HW6M Problem 1, textbook pp. 114–15, GP5W Problem 3, Exam 2 Q5
» Fibonacci numbers (definition)
  – *Problems:* HW7M Problem 1, GP6M Problem 1, Exam 2 Q2
» Graph theory
  – $\sum_{v\in V} \deg_G(v) = 2|E|$
    * *Problems:* HW8M Problem 1, GP7M Problems 1 and 2, Exam 2 Q4
  – Paths, cycles, theorem about existence of closed Eulerian walks
    * *Problems:* HW8W Problem 1, GP7W Problem 1, Exam 2 Q4
  – Trees
    * Definition: connected acyclic graph
    * Trees with n vertices have $n-1$ edges
    * Cayley's formula: the number of labeled trees with n vertices is n^{n-2}
      · Joyal's proof of Cayley's formula: https://people.reed.edu/~davidp/113/handouts/joyal.pdf
      · *Problems:* HW10M Problem 1, GP9M
» Catalan numbers
  – Catalan number C_n counts unlabelled full binary trees with $n+1$ leaves
  – Catalan recurrence: $C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$
    * *Problems:* GP7F Problems 1 and 2
  – Closed formula: $C_n = \frac{1}{n+1}\binom{2n}{n}$

  - Structures counted by Catalan numbers: bijections between between Dyck paths, full binary trees, balanced parenthesizations, and full parethesizations of letters (https://people.reed.edu/~davidp/113/handouts/catalan_bijections.pdf)
    * *Problems:* HW9W Problem 1

» Probability theory
  - Basic objects: sample space, events, probability distribution
    * *Problems:* HW11W, GP9W Problems 1 and 2
  - Uniform distribution on a finite sample space
  - Independent events
    * *Problems:* GP9F Problems 1 and 2
  - Conditional probability $P(A|B)$
  - Bayes' Law: $P(B|A) = P(A|B)P(B)/P(A)$
  - Law of Total Probability: $P(A) = \sum_i P(A|B_i)P(B_i)$ for $\{B_i\}$ a partition
  - Random variables
  - Expected value $E(X)$
    * *Problems:* HW11M
  - Linearity of expected value: $E(cX + Y) = cE(X) + E(Y)$
    * *Problems:* GP10W Problem 1
  - Special types of random variables: Bernoulli, binomial, indicator
    * *Problems:* HW10M Problem 2, HW10W Problem 1, GP10M, GP10F

» Number theory
  - Divisibility
    * *Problems:* HW11F Problem 1 (using the fundamental theorem of arithmetic), HW12M Problem 1, GP11M
  - Prime numbers
  - Fundamental theorem of arithmetic (unique prime factorization)
  - Infinitude of primes
  - Prime number theorem: $\pi(n) \sim n/\log n$
  - Fermat's little theorem: $p|a^p - a$
  - Greatest common divisors and Euclidean algorithm
    * *Problems:* HW12F Problems 1 and 3
  - Congruences
    * *Problems:* GP12W Problems 1–4, GP12F
  - Multiplicative inverses modulo n
  - Euler ϕ function
    * $\phi(n) = |\{r \in \mathbb{N} \mid r < n \text{ and } \gcd(r, n) = 1\}|$
    * Euler's formula: $\phi(n) = n(1 - 1/p_1)(1 - 1/p_2)\dots(1 - 1/p_k)$ for n with prime factors $p_1, \dots, p_k$
    * Euler's theorem: if $\gcd(a, n) = 1$, then $a^{\phi(n)} = 1 \bmod n$
      · *Problems:* GP13M Problem 1
  - Sunzi's theorem
    * *Problems:* HW13F Problem 2, GP13W Problems 1–4

Links to compiled lists of homework and group problems:

» https://people.reed.edu/~davidp/113/handouts/all-hw.pdf
» https://people.reed.edu/~davidp/113/handouts/all-groups.pdf
» https://people.reed.edu/~davidp/113/handouts/all-group-sols.pdf