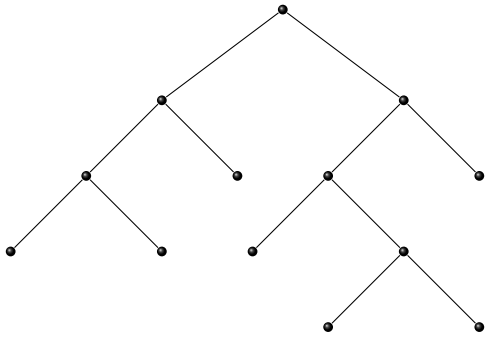
Math 113 Catalan bijections

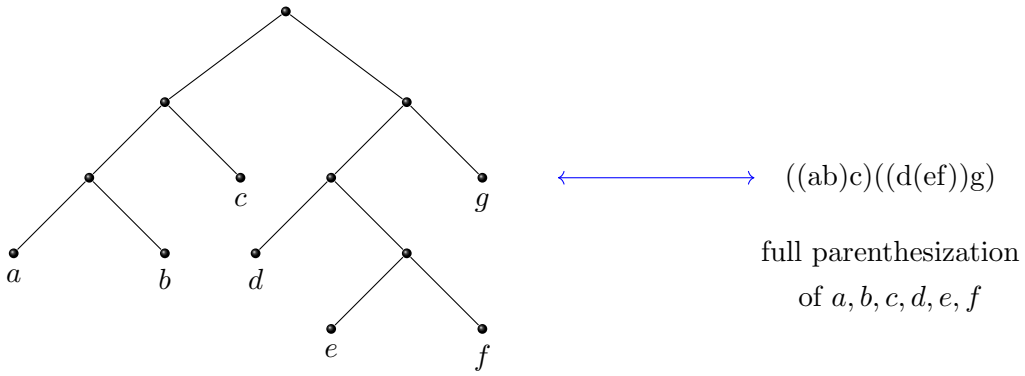
In our readings and group work, we have developed bijections between the following "Catalan objects":

- Full binary trees on $n+1$ leaves.
- Full parenthesizations of $n+1$ letters $a, b, c, \ldots$ ("full" means each multiplication is binary, i.e., involves two factors)
- Balanced parenthetical expressions with n pairs of ().
- Dyck paths of length $2n$.

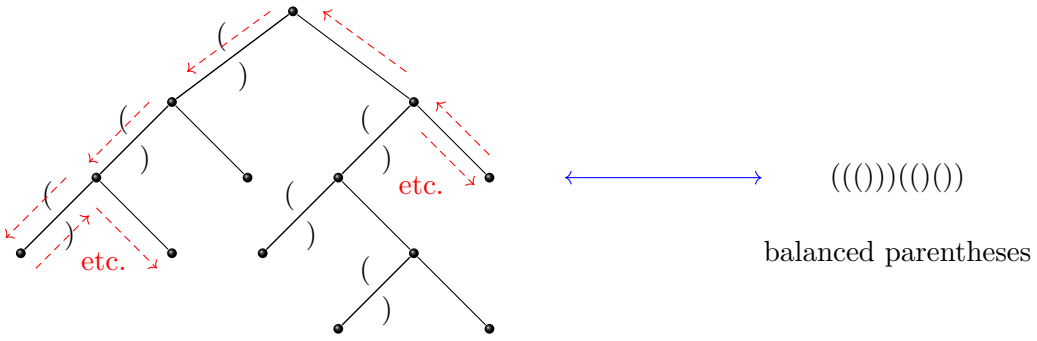
Our running example will be the full binary tree pictured below:



Full binary tree to parenthesizations of $n+1$ letters. Labeling the leaves of the tree from left to right make this bijection clear:

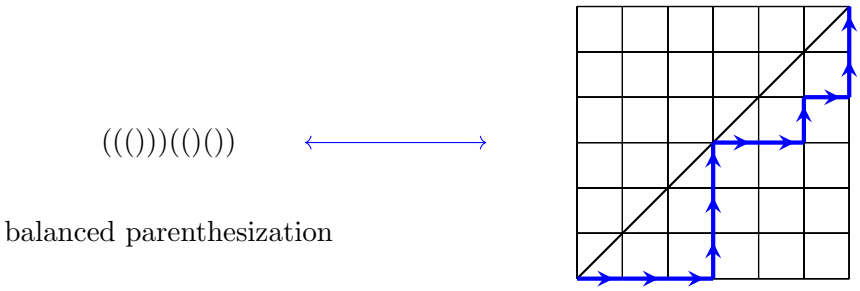


Full binary tree to balanced parenthesization. To form the corresponding balanced parenthesization, we label each left edge with a "(" on its left and a ")" on its right. We then take a clockwise trip around the tree, hugging close to the edges and reading off the labels (the dashed line gives a hint of the path):



full binary tree

Balanced parenthesization to Dyck path. The correspondence between balanced parenthesizations and Dyck paths is easy: convert "(" to "r" (a right/east step) and ")" to "u" (an up/north step).



Dyck path