MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE WEDNESDAY WEEK 7

Problem 1. Let $\phi := \frac{1+\sqrt{5}}{2}$ and $\overline{\phi} := \frac{1-\sqrt{5}}{2}$. Use induction to prove that

$$F_n = \frac{1}{\sqrt{5}}\left(\phi^n - \overline{\phi}^n\right)$$

for $n \geq 0$. [Hints: For the base of induction, you need to check both $n = 0$ and $n = 1$. Using the quadratic equation, one may easily check that ϕ and $\overline{\phi}$ are the two solutions to the equation $x^2 = x + 1$. Thus, these numbers satisfy the relations

$$\phi^2 = \phi + 1 \quad \text{and} \quad \overline{\phi}^2 = \overline{\phi} + 1.$$

Using the symbols ϕ and $\overline{\phi}$ as much as possible along with these identities will simplify your work.]

Problem 2. Define a sequence of Fibonacci-like numbers L_n by

$$L_n = \begin{cases} 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ L_{n-1} + L_{n-2} & \text{if } n > 1. \end{cases}$$

(a) Compute $L_0, L_1, \ldots, L_{10}$.

(b) Use induction to show that

$$L_n = F_{n-1} + F_{n+1}$$

for $n \geq 0$. (There are two base cases, again. Also, recall that in the Course Log, we established that $F_{-1} = 1$.)

(c) There is a closed form for L_n similar to that for the Fibonacci numbers:

$$L_n = \phi^n + \overline{\phi}^n,$$

where ϕ and $\overline{\phi}$ are as in Problem 1. Use these closed forms to prove that

$$F_{2n} = F_n L_n.$$