MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE MONDAY WEEK 7

Problem 1. Use the recurrence relation defining Fibonacci numbers to prove that

$$F_{2n+1} = 3F_{2n-1} - F_{2n-3}$$

for $n \geq 1$. (This gives us a way to compute Fibonacci numbers of odd index without computing those with even index!)

Problem 2. Our text uses induction to show that

$$F_{2n+1} = F_n^2 + F_{n+1}^2. \tag{1}$$

You are now asked to give a combinatorial proof using tilings of checkerboards. Let a_n be the number of ways of tiling a $2 \times n$ checkerboard with 2×1 dominoes. In class, we found that $a_n = F_{n+1}$ (see Problem 18.1 from Day 15 of our Course Log and its solution appearing at the end of the Course Log).

Rewrite equation (1) in terms of appropriate a_i and prove the resulting (equivalent) formula by counting tilings of a $2 \times 2n$ checkerboard. (Hint: Our checkboard has two halves, each of size $2 \times n$. Consider how dominoes in a tiling behave at the middle where these two halves meet. There are only two possibilities!)