MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE FRIDAY WEEK 13

Problem 1.

(a) For $n = 10, 11$, and 12. List the fractions

$$\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}$$

after reducing each to lowest terms (canceling common factors in the numerator and denominator).

(b) Let n be a natural number and consider the quantity

$$\psi(n) = \sum_{d|n} \phi(d)$$

which is the sum of the values $\phi(d)$ where d ranges through all the positive divisors of n. What is $\psi(n)$? (Experiment, formulate a conjecture, and prove it.) Your solution should consist of a precise statement of your conjecture and a proof. The proof does not need to be elaborate. It can just be a statement of the general relevant phenomenon you observe in part (a).

Problem 2. Use Sunzi's Theorem to efficiently compute the congruence class of 17^2 modulo 35. (Show your work: What is the value to $\overline{17}^2 \in \mathbb{Z}/5\mathbb{Z}$? and in $\mathbb{Z}/7\mathbb{Z}$? Use Sunzi's Theorem to push these results back into $\mathbb{Z}/35\mathbb{Z}$.)