

MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE FRIDAY WEEK 12

Problem 1. Use the Euclidean algorithm to compute the following (showing your work):

- (a) $\gcd(20, 45)$ (b) $\gcd(247, 299)$ (c) $\gcd(51, 897)$.

Problem 2. Use the Euclidean algorithm to compute the gcd of 198 and 168 and find integers m and n such that

$$\gcd(198, 168) = 198m + 168n.$$

Use the method described on p. 104 of our text, perhaps more clearly explained here:

https://www.youtube.com/watch?v=oXRT_TNoBaM

Problem 3.

- (a) Show that if n is positive integer of the form $4k + 3$ for some integer k , then n is not a perfect square. (Hint: Suppose $n = m^2$. We can then write $m = 4q + r$ for some $r \in \{0, 1, 2, 3\}$. Consider the remainders of the quantities $(4q)^2$, $(4q+1)^2$, $(4q+2)^2$, and $(4q+3)^2$ upon division by 4.)
- (b) Show that no integer in the sequence

$$11, \quad 111, \quad 1111, \quad 11111, \quad \dots$$

is a perfect square. [Hint: Use the fact that $111 \dots 1111 = 111 \dots 1108 + 3$.]