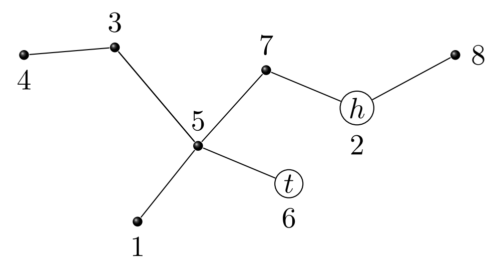
MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE MONDAY WEEK 10

Problem 1. The following is practice on understanding Joyal's proof of Cayley's formula. Recall that Cayley's formula says that the number of trees on n vertices is $T_n := n^{n-2}$. Joyal proves this by giving a combinatorial argument that $n^2 T_n = n^n$. The left-hand side counts "vertebrates", which consist of a tree on n-vertices and a choices of special tail and head vertices. The right-hand side counts functions $\underline{n} \to \underline{n}$. For the following problems use Joyal's bijection between these two set of objects, as described in our handout and in class.

(a) Find the function associated with the following vertebrate:



(b) Find the vertebrate associated with the following function:

i	1	2	3	4	5	6	7	8
$f(i)$	3	2	4	1	7	1	2	6

(c) Describe the vertebrates arising from constant functions $\underline{n} \to \underline{n}$ (do not forget to specify the tail and head). (A *constant function* is a function whose image consists of a single element.)

Problem 2. Recall that a standard 52-card deck of cards consists of four suits: clubs, spades, hearts, and diamonds (where clubs and spades are black, and hearts and diamonds are red). Each suit has 13 denominations: ace, two, three,. . . , ten, jack, queen, and king. You are dealt five cards from a shuffled standard 52-card deck. What are the probabilities of receiving each of the following? (Provide explanations.)

(a) A flush (five cards of the same suit, no restriction on the denominations).

(b) A full house (three cards of one denomination and two of another; for example, three kings and two sevens is a full house).