

Math 113 Group Problems for Wednesday, Week 13

PROBLEM 1. Does Sunzi's theorem still hold if we drop the requirement that the n_i are relatively prime? Prove your assertion or provide a counterexample.

PROBLEM 2. A group of 17 people stack their books in 11 piles of equal size, each containing more than one book, and an additional pile containing 6 books. They collect the books and this time stack them into 17 equally-sized piles, with no left over. What is the smallest number of books they could have had? [Hint: -3 is the multiplicative inverse of 11 modulo 17.]

PROBLEM 3. Find *all* solutions $x \in \mathbb{Z}$ to the system of congruences

$$\begin{aligned}x &= 2 \pmod{4} \\x &= 3 \pmod{5} \\x &= 4 \pmod{9}.\end{aligned}$$

PROBLEM 4. Find all integers x, y such that

$$\begin{aligned}2x + 5y &= 4 \pmod{11} \\x + 3y &= 7 \pmod{11}.\end{aligned}$$

PROBLEM 5. Show there are no integer solutions to the equation

$$x^4 - 125x^3 - 75x^2 + 5x = 123456789.$$