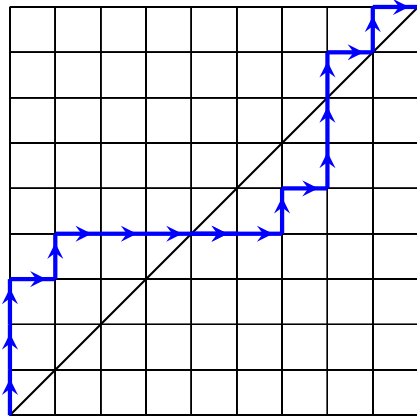


Math 113 Group Problems for Monday, Week 8

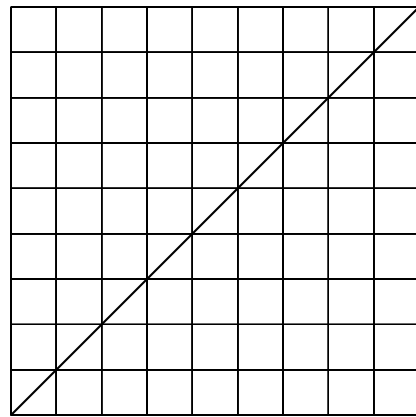
PROBLEM 1. Illustrate the bijection $E_i \rightarrow E_{i+1}$.

$BrAuC$

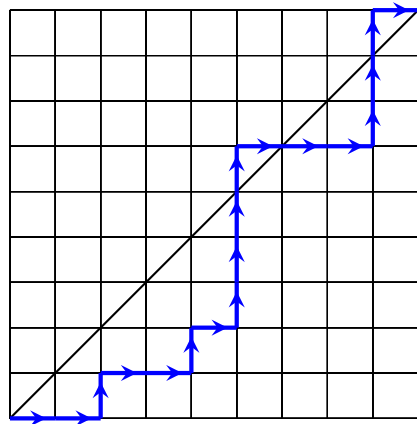


Exceedance =

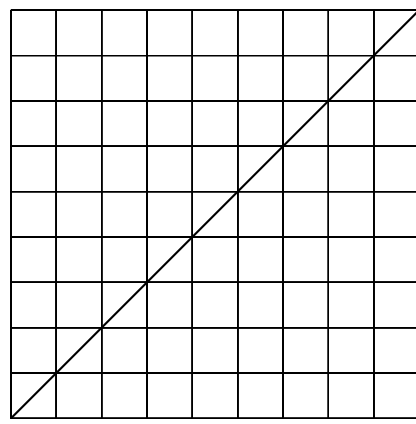
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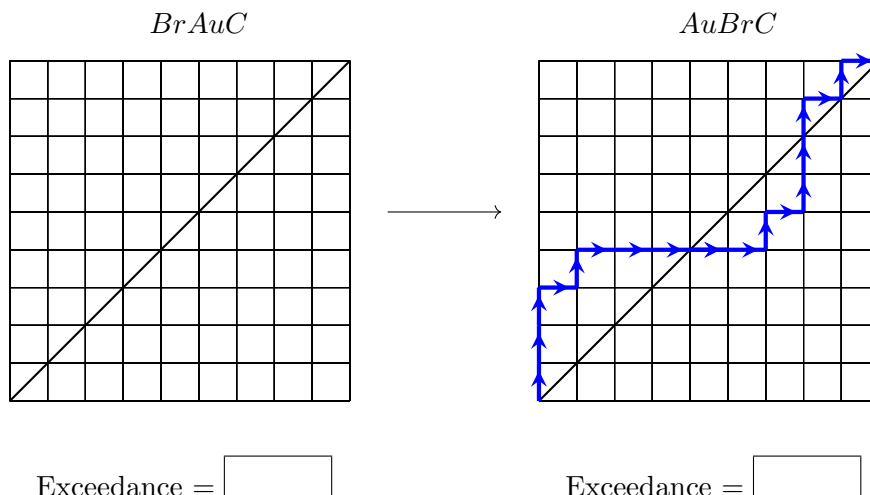


Exceedance =



Exceedance =

PROBLEM 2. Illustrate the inverse of the bijection $E_i \rightarrow E_{i+1}$.



PROBLEM 3. In our group problems for last time, you saw that C_n counts the number of expressions consisting of n balanced parentheses $()$.

- (a) Describe a bijection between Dyck paths and balanced parentheses.
- (b) Apply your bijection to the two Dyck paths on the left on page 1.
- (c) What is the Dyck path associated with $((()))()$?

PROBLEM 4. In our group problems for last time, we saw a bijection between full binary trees with $n + 1$ leaves and expressions consisting of n balanced parentheses. (Label each left edge in a full binary tree with “(” on the left and each right edge with “)” on the right. Get on your bicycle and ride clockwise around the full binary tree, starting at the root and staying close to the edges. Read off the parentheses as you pass by.) Combining this bijection with the one between balanced parenthetical expressions and Dyck paths, we get a bijection between full binary trees and Dyck paths. What are the full binary trees associated with the two Dyck paths on the left on page 1?