

L^AT_EX SAMPLE

GOTTFRIED LEIBNIZ

Theorem 1. *The derivative of a function f at c is given by*

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

provided this limit exists.

Proof. By definition, we know that

$$(1) \quad f'(c) = \lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}.$$

Let $x = c + \Delta x$. Then $x \rightarrow c$ as $\Delta x \rightarrow 0$. So, replacing $c + \Delta x$ by x and Δx by $x - c$ in (1), we find that

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

as desired. □