MATH 113: DISCRETE STRUCTURES
EXAM 2 REVIEW

For the second exam, you are responsible for the content covered in class, our reading assignments, and video lectures through Monday, September 25. The emphasis of the exam will strongly skew towards content covered after the first exam. Topics include: binomial magic and Pascal's triangle, derangements, recurrence, the Fibonacci numbers, mathematical induction, and discrete probability (distributions, independence, Bayes' theorem, total probability, random variables, expectation and linearity thereof, binomial and geometric distributions).

The exam is an in-class, 50-minute exam. You may bring one two-sided sheet of notes (US letter size or A4 paper). No other resources (including books, other notes, electronic aids, or consultation) are permitted.

The following problems are for practice and are intended to help you review for the exam. They are not to be turned in and will not be graded, though I am happy to provide feedback if you have questions about a solution.

One of these problems will appear on the exam, but most of these problems are *harder* than exam problems.

Problem 1. For which n is $\binom{2n}{n} < 4^n$? Prove your assertion. (*Bonus*: Give a really slick and short proof.)

Problem 2. How many subsets of $\underline{n}$ are larger than their complement?

Problem 3. Find a closed formula for a_n where $a_0 = 2$, $a_1 = 3$, and $a_n = 6a_{n-1} - 9a_{n-2}$ for $n \geq 2$.

Problem 4. Suppose A and B are two events with $P(A) = 0.5$, $P(A \cup B) = 0.8$.

(a) For what values of $P(B)$ would A and B be mutually exclusive?
(b) For what values of $P(B)$ would A and B be independent?

Problem 5. A digital communications system consists of a transmitter and a receiver. During each short transmission interval the transmitter sends a signal to be interpreted as a 0 or a 1. At the end of each interval, the receiver makes its best guess at what was transmitted. For $i = 0, 1$, let T_i be the event that the transmitter sends i and let R_i be the event that the receiver concludes that i was sent. Assume that $P(R_0|T_0) = 0.99$, $P(R_1|T_1) = 0.98$, and $P(T_1) = 0.5$.

(a) What is the probability of a communication error given R_1?
(b) What is the overall probability of a communication error?

Problem 6. A building has 10 floors above the ground floor and no basement. If 12 people get into the elevator at the ground floor, and each chooses a non-ground floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop?

Problem 7 (Indicator variables and inclusion-exclusion). Let I_A be the indicator variable for an event $A \subseteq S$. Recall that $I_A(s) = 1$ if $s \in A$ and otherwise $I_A(s) = 0$; further recall that $E(I_A) = P(A)$.

(a) Prove that $I_{A^c} = 1 - I_A$.
(b) Prove that $I_{A \cap B} = I_A I_B$.
(c) Prove that for any $A_1, A_2, \ldots, A_n \subseteq S$,

$$I_{A_1 \cup A_2 \cup \cdots \cup A_n} = 1 - (1 - I_{A_1})(1 - I_{A_2}) \cdots (1 - I_{A_n}).$$

(d) Expand the product in the formula from (c) and use linearity of expectation to derive the inclusion-exclusion formula.

Problem 8. Suppose you pick people at random and ask them their birthday. Let X be the number of people you have to question until you find a person who was born in November. What is $E(X)$ (assuming birthdays are distributed uniformly randomly, and ignoring the existence of leap days)?