MATH 113: DISCRETE STRUCTURES
HOMEWORK FOR FRIDAY WEEK 9

Problem 1 (Indicator variables). Given an event $A \subseteq S$ and $s \in S$, let $I_A(s) = 1$ if $s \in A$, and let $I_A(s) = 0$ if $s \notin A$. This defines a random variable I_A called the *indicator variable* of A. Prove that $E(I_A) = P(A)$.

Problem 2 (The number of surviving rabbits). There are n hunters and n rabbits in a field. Each hunter aims their gun at exactly one rabbit, and then all of the hunters fire simultaneously. Alas – every hunter is a perfect shot and any rabbit which was aimed at dies. In this problem, you will determine how many rabbits are expected to survive.

(a) Translate this problem in the language of functions.
(b) For $1 \leq i, j \leq n$, what is the probability that the j-th hunter shoots the i-th rabbit?
(c) For $1 \leq i \leq n$, let A_i be the event that the i-th rabbit survives. What is $P(A_i)$?
(d) Let X be the total number of surviving rabbits. Use indicator variables to express X as a sum of random variables and determine $E(X)$.
(e) Use a calculator or computer to approximate $E(X)/n$, the expected fraction of rabbits to survive, when n is very large. *Bonus*: Determine the exact value of $\lim_{n\to\infty} E(X)/n$.