

MATH 113: DISCRETE STRUCTURES
HOMEWORK FOR WEDNESDAY WEEK 8

Problem 1 (DM:EB 5.2.3). Consider an experiment with sample space S repeated n times ($n \geq 2$). Let $s \in S$. Let A be the event that the first outcome is s , and let B be the event that the last outcome is s . Prove that A and B are independent (relative to the uniform probability distribution).

Problem 2. Events A_1, A_2, \dots, A_k are *fully independent* if for all subsets $I \subseteq \{1, 2, \dots, k\}$,

$$P\left(\bigcap_{i \in I} A_i\right) = \prod_{i \in I} P(A_i).$$

(The indexed intersection takes the intersection of all the A_i where i ranges through I ; the indexed product takes the product of all the $P(A_i)$ where i ranges through I .) Construct three events A, B, C which are pairwise independent but not fully independent. (I.e., $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, and $P(B \cap C) = P(B)P(C)$, but $P(A \cap B \cap C) \neq P(A)P(B)P(C)$.) *Bonus:* What is the smallest sample space for this problem?