MATH 113: DISCRETE STRUCTURES
MONDAY WEEK 4 HANDOUT

Problem 1. Prove the identity

$$1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + (n-1) \cdot 2 + n \cdot 1 = \binom{n+2}{3}$$

for $n \geq 2$ both by induction and combinatorially.

Problem 2. Give a combinatorial proof that for $1 \leq k \leq n$,

$$n^k \leq \binom{n}{k} k^k.$$

Observe that it follows that

$$\left(\frac{n}{k}\right)^k \leq \binom{n}{k}.$$

Problem 3. Use induction to prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

for $n \geq 1$.

Problem 4. Use induction to prove that a convex n-gon has $n(n-3)/2$ diagonals.

Problem 5. Use induction to prove that

$$\binom{2n}{n} < 2^{2n-2}$$

for $n \geq 5$. Can you give a combinatorial argument as well?