MATH 113: DISCRETE STRUCTURES
MONDAY WEEK 10 HANDOUT

First, our promised proof of Markov's inequality. Recall that we have a nonnegative random variable X and a real number $t \geq 1$. For convenience, set $a = tE(X)$. Then

$$\begin{aligned}
E(X) &= \sum_{y \in X(S)} yP(X = y) && \text{(by definition)} \\
&= \sum_{y<a} yP(X = y) + \sum_{y \geq a} yP(X = y) && \text{(splitting the sum into two pieces)} \\
&\geq \sum_{y \geq a} yP(X = y) && \text{(since everything is } \geq 0) \\
&\geq \sum_{y \geq a} aP(X = y) && \text{(since } y \geq a \text{ for this piece)} \\
&= a \sum_{y \geq a} P(X = y) && \text{(factoring)} \\
&= aP(X \geq a) && \text{(partition of the event } X \geq a).
\end{aligned}$$

Dividing by a (which is nonnegative) we get

$$P(X \geq a) \leq \frac{E(X)}{a}.$$

Since $a = tE(X)$, this is equivalent to

$$P(X \geq tE(X)) \leq \frac{1}{t}.$$

Let's now move on to some problem-solving with binomial and geometric distributions.

Problem 1. With a small group, roll a pair of dice twelve times. Record the first roll on which you roll doubles and also the total number of doubles that you roll and report these numbers to the instructor. What is the expected number of doubles in twelve rolls? How long should it take to roll doubles? How do these numbers compare with the class's statistics?

Problem 2. An airline has sold 205 tickets for a flight that can hold 200 passengers. Each ticketed person, independently, has a 5% chance of not showing up for the flight. What is the probability that more than 200 people will show up for the flight?

Problem 3. If the same airline consistently oversells the flight from Problem 2 at the same rate, how many flights until we expect more ticketed passengers to show up than there are seats.