

**MATH 113: DISCRETE STRUCTURES**  
**HOMEWORK DUE MONDAY WEEK 11**

*Problem 1 (DM:EB 5.2.3).* Consider an experiment with sample space  $S$  repeated  $n$  times ( $n \geq 2$ ). Let  $s \in S$ . Let  $A$  be the event that the first outcome is  $s$ , and let  $B$  be the event that the last outcome is  $s$ . Prove that  $A$  and  $B$  are independent (relative to the uniform probability distribution).

*Problem 2.* Events  $A_1, A_2, \dots, A_k$  are *fully independent* if for all subsets  $I \subseteq \{1, 2, \dots, k\}$ ,

$$P\left(\bigcap_{i \in I} A_i\right) = \prod_{i \in I} P(A_i).$$

(The indexed intersection takes the intersection of all the  $A_i$  where  $i$  ranges through  $I$ ; the indexed product takes the product of all the  $P(A_i)$  where  $i$  ranges through  $I$ .) Construct three events  $A, B, C$  which are pairwise independent but not fully independent. (I.e.,  $P(A \cap B) = P(A)P(B)$ ,  $P(A \cap C) = P(A)P(C)$ , and  $P(B \cap C) = P(B)P(C)$ , but  $P(A \cap B \cap C) \neq P(A)P(B)P(C)$ .) *Bonus:* What is the smallest sample space for this problem?