MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE WEDNESDAY WEEK 14

Problem 1. Use Sunzi's Theorem to efficiently compute the congruence class of 17^2 modulo 35. (Show your work: What is the value of $[17]^2$ in $\mathbb{Z}/5\mathbb{Z}$ and in $\mathbb{Z}/7\mathbb{Z}$? Use Sunzi's Theorem to push these results back into $\mathbb{Z}/35\mathbb{Z}$.)

Problem 2. By following the below steps, you will ultimately prove that every positive integer n, there exist relatively prime integers $k_0, \ldots, k_n > 1$ such that $k_0 \cdots k_n - 1$ is the product of two consecutive integers.

(a) Emulate Euclid's proof of the infinitude of primes to show that there are infinitely many primes that divide some number of the form $t^2 + t + 1$, $t \in \mathbb{N}$. In other words, show that

$$P = \{p \text{ prime} \mid p \text{ divides } t^2 + t + 1 \text{ for some } t \in \mathbb{N}\}$$

is infinite.

(b) Use Sunzi's theorem and part (a) to show that for all $n \geq 0$, there exists $t \in \mathbb{N}$ such that $t^2 + t + 1$ is divisible by some product $p_0 \cdots p_n$ of n distinct primes.

(c) Given t and $p_0, \ldots, p_n$ as in (b), conclude that there exist relatively prime integers $k_0, \ldots, k_n$ such that $k_0 \cdots k_n - 1$ is the product of two consecutive integers.