MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE MONDAY WEEK 14

Problem 1. Find, with proof, the remainder of $9^{1260126012601260128}$ upon division by 14.

Problem 2.

(a) For $n = 10, 11$, and 12. List the fractions
$$\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}$$
after reducing each to lowest terms (canceling common factors in the numerator and denominator).

(b) Let n be a natural number and consider the quantity
$$\psi(n) = \sum_{d|n} \varphi(d)$$
which is the sum of the values $\varphi(d)$ where d ranges through all the positive divisors of n. What is $\psi(n)$? (Experiment, formulate a conjecture, and prove it.) Your solution should consist of a precise statement of your conjecture and a proof. The proof does not need to be elaborate. It can just be a statement of the general relevant phenomenon you observe in part (a).

Problem 3. There are integers n such that -1 has a square root in $\mathbb{Z}/n\mathbb{Z}$. To test this out, for each $n \in \{2, 3, \ldots, 13\}$, find all solutions $x \in \{0, 1, \ldots, n-1\}$ to the equation
$$x^2 \equiv -1 \pmod{n}.$$
You do not need to show your work. It may help to note that $-1 \equiv n - 1 \pmod{n}$.

Problem 4. Find all solutions $x \in \{0, 1, \ldots, n-1\}$ to the congruence $3x^2 - x + 1 \equiv 0 \pmod{n}$ for $n = 8$ and for $n = 9$. You do not need to show your work (but double-check your results!).