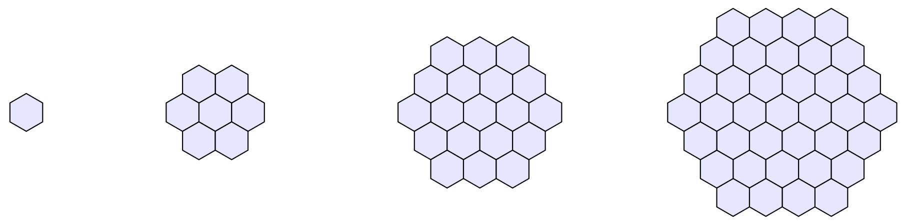
MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE FRIDAY WEEK 14

Problem 1. Let h_n be the number of hexagons in a central packing of hexagons with $n+1$ layers. The first few values are pictured below:



$h_0 = 1$ $\quad$ $h_1 = 7$ $\quad$ $h_2 = 19$ $\quad$ $h_3 = 37$

Recall the first difference operator $\Delta[h]_n = h_{n+1} - h_n$.

(a) Notice that the pictures above nest in each other about the origin. Use that fact to draw pictures for the first differences $\Delta[h]$ and use the pictures to determine the sequence of second differences $\Delta^2[h]$.

(b) **Using the theory of difference operators from the text** and the results above, compute a polynomial $p(n)$ such that $p(n) = h_n$ for all $n \geq 0$.

Problem 2. Our text uses induction to show that

$$F_{2n+1} = F_n^2 + F_{n+1}^2. \tag{1}$$

You are now asked to give a combinatorial proof using tilings of checkerboards. Let a_n be the number of ways of tiling a $2 \times n$ checkerboard with 2×1 dominoes. At the end of the section on induction, our text shows that $a_n = F_{n+1}$.

Rewrite equation (1) in terms of appropriate a_i and prove the resulting (equivalent) formula by counting tilings of a $2 \times 2n$ checkerboard. (Hint: Our checkboard has two halves, each of size $2 \times n$. Consider how dominoes in a tiling behave at the middle where these two halves meet. There are only two possibilities: there is a pair of dominoes straddling the two halves, or no domino straddles the center. Given that each half of our chessboard contains an even number of dominoes, it is impossible for only one domino to straddle the center.)