MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE WEDNESDAY WEEK 10

Problem 1. You have three coins. Two of the coins are fair: when flipped they are equally likely to land heads or tails. One coin, however, is weighted somehow so that its probability of landing heads is $3/4$.

(a) Choose one of the three coins uniformly at random and flip it. What is the probability the result is heads? For your solution, number the coins $1, 2, 3$ with coin 3 being the weighted one, and let A_i denote the event that coin i was chosen. Apply the generalized law of total probability (Theorem 155 our text).

(b) Choose one of the three coins at random and flip it. It lands heads. What is the probability that you chose the weighted coin? (Hint: Bayes' law.)

Problem 2. Reconsider the Monty Hall problem as stated in the group problems for Friday, Week 10, but where the game show has a bias for where it places the car so that $P(A) = 0.4$, $P(B) = 0.3$, and $P(C) = 0.3$. (In advance of your turn on the show, suppose that you studied taped shows to determined these propensities.) As in the group problems, let M_A, M_B, and M_C denote the events that "the host opens door A", "door B", and "door C", respectively.

(a) Show that no matter which door you pick, it makes sense to switch.

(b) Which door should you pick?

(c) What are your chances of eventually winning the car if you make that pick?

Note:

» The rules for Monty are the same: if you pick the door with the car, then Monty chooses between the remaining doors each with probability $1/2$, otherwise, Monty has only one choice: pick the door without the car.

» Up to symmetry, you just need to consider three cases: (i) you pick door A and Monty picks door B, (ii) you pick door B and Monty picks door A, and (iii) you pick door B and Monty picks door C. (If the all three of $P(A)$, $P(B)$, and $P(C)$ were different, there would be six cases to think about.)