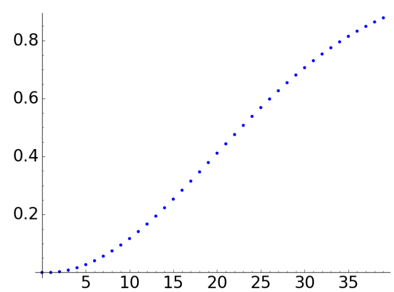
**MATH 113: DISCRETE STRUCTURES**
**HOMEWORK DUE MONDAY WEEK 10**

*Problem* 1. Suppose a fair coin is flipped $n > 1$ times. We record the result at a string of length $n$ in the letters $H$ and $T$. For instance, if $n = 3$, then $HTT$ says that the first flip was heads and the next two were tails. Our sample space $S$ is, thus, the set of strings of length $n$ consisting of the letters $H$ and $T$, upon which we place the uniform distribution. Let $A$ be the event that the first flip is heads, and let $B$ be the event that the last flip is heads. Prove that these events are independent by computing the relevant probabilities and using the definition of independence.

*Problem* 2. This is the famous birthday problem. Suppose there are $n$ people in a room. For simplicity, we will say that there are $365$ possible birthdays (i.e., we will ignore leap years) and that each day is equally likely to be a birthday. To create the sample space, $S$, number the people from $1$ to $n$, then the possible outcomes are the sequences of length $n$ where the $i$-th element of the sequence is a possible birthday for person $i$. Let $B$ be the event that at least two people in the room share a birthday.

(1) What is $|S|$?
(2) Consider the complementary event $B^c$, i.e., the event that no two people have the same birthday. Give an expression for $B^c$, and use it to find the probability $P(B^c)$. (The result will depend on $n$.)
(3) Use the previous result to give an expression for $P(B)$.
(4) A plot of $P(B)$ as a function of $n$ looks like this:




Use a calculator of some sort to give decimal approximations, accurate to three decimal places, for $P(B)$ when $n = 22$ and $n = 23$. You should see that if the room contains at least 23 people, then the probability two people share the same birthday is more than one half.