

**MATH 113: DISCRETE STRUCTURES**  
**HOMEWORK DUE FRIDAY WEEK 6**

*Problem 1.* A graph is  $k$ -regular if each of its vertices has degree  $k$ .

- (a) Draw a 3-regular graph on 6 vertices.
- (b) Prove that there are no 3-regular graphs on 5 vertices.

*Problem 2.* If  $G$  is a graph and  $e$  is an edge of  $G$ , define  $G - e$  to be the graph obtained from  $G$  by removal of  $e$  (but not the endpoints of  $e$ ). Recall that to say a graph is *connected* means that every pair of its vertices can be connected by a path.

- (a) Give an example of a connected graph  $G$  with an edge  $e$  such that  $G - e$  is not connected.
- (b) Suppose  $G$  is a connected graph and  $e$  is an edge of  $G$  that is part of a cycle. Prove that removal of  $e$  does not disconnect the graph. Your proof is required to start with the line: "Let  $u$  and  $v$  be vertices of  $G$ ." It should then show there must be a path in  $G - e$  connecting  $u$  and  $v$ .