MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE MONDAY WEEK 5

Note. See the next page for a model proof by induction. Try to emulate it in your own work.

Problem 1. Use induction to prove that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 1$.

Problem 2. Suppose that n lines in the plane are drawn in such a fashion that no two are parallel and no three intersect in a common point. Prove that the plane is divided into precisely $\frac{n(n+1)}{2} + 1$ regions by the lines.

A typical induction proof

Proposition. For $n \geq 1$,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Proof. We will prove this by induction. First note that the statement holds when $n = 1$:

$$1 = \frac{1(1+1)}{2}.$$

Next, suppose the statement holds for some $n \geq 1$:

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

It follows that

$$\begin{aligned} 1 + 2 + \cdots + n + (n+1) &= \frac{n(n+1)}{2} + (n+1) \\ &= \frac{n(n+1) + 2(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2}, \end{aligned}$$

and the result then holds for $n + 1$, too. Hence, the statement holds for all $n \geq 1$ by induction. □

Note:

» The first sentence of the proof is obligatory. The reader needs to know you are about to give a proof by induction.

» It is good to explicitly state your induction hypothesis. In the above proof, it is the sentence starting "Next, suppose ..." You are not claiming this statement is true! Your argument will be that *if* this statement is true, then something good happens (namely, the statement also holds for the case $n + 1$).

» Notice the easy-to-follow linear arrangement of equations following "It follows that". When you have a string of calculations, please try to use a similar form.