MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE MONDAY WEEK 4

Problem 1. Suppose that $f\colon A \to B$ is a surjective function. Define a relation $\asymp_f$ on A so that $a \asymp_f b$ if and only if $f(a) = f(b)$.

(a) Prove that $\asymp_f$ is an equivalence relation.
(b) Determine the number of equivalence classes under $\asymp_f$.

Problem 2. Suppose that we are playing a game in which we roll three six-sided dice (with sides labeled $1, 2, \ldots, 6$). Declare two rolls equivalent if their sums match. (Formally, a roll can be thought of as an ordered 3-tuple (a, b, c) where $a, b, c \in \{1, \ldots, 6\}$, and our relation is $(a, b, c) \sim (a', b', c')$ if and only if $a + b + c = a' + b' + c'$).)

(a) Prove that this is indeed an equivalence relation.
(b) Determine the number of equivalence classes.
(c) Are all of the equivalence classes of the same size?

Template for proving a relation is an equivalence relation.

Theorem. Define a relation $\sim$ on a set A by blah, blah, blah. Then $\sim$ is an equivalence relation.

Proof. *Reflexivity.* For each $a \in A$, we have $a \sim a$ since blah, blah, blah. Therefore, $\sim$ is reflexive.
Symmetry. Suppose that $a \sim b$. Then, blah, blah, blah. It follows that $b \sim a$. Therefore $\sim$ is symmetric.
Transitivity. Suppose that $a \sim b$ and $b \sim c$. Since blah, blah, blah, it follows that $a \sim c$. Therefore, $\sim$ is transitive.
Since $\sim$ is reflexive, symmetric, and transitive, it follows that $\sim$ is an equivalence relation. $\square$