PROBLEM 1. Consider the sequence a_0, a_1, \ldots defined by the recurrence

$$a_0 = 0$$
, $a_1 = 1$, and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \ge 2$.

- (i) Write out the terms of (a_n) until you get to 2059.
- (ii) Check that for $a \neq b$,

$$\frac{x}{(1-ax)(1-bx)} = \frac{1}{a-b} \left(\frac{1}{1-ax} - \frac{1}{1-bx} \right).$$

(iii) In the text, we used generating functions to find a closed form for the Fibonacci numbers. Apply a similar procedure to $f(x) = a_0 + a_1x + \cdots + a_nx^n + \cdots$, the generating function for (a_n) , to find a closed form for (a_n) .

PROBLEM 2. Let $f(x) = \sum_{i=0}^{\infty} b_i x^i$ be the generating function for the sequence b_0, b_1, \ldots

- (i) Let g(x) = (1 x)f(x). Then $(g(x) b_0)/x$ is the generating function for which sequence?
- (ii) Let $h(x) = \frac{f(x)}{1-x}$. Then h(x) is the generating function for which sequence?
- (iii) Apply the previous result to h(x) = 1/(1-x) to find the sequence whose generating function is $1/(x-1)^2$.
- (iv) Find the sequence whose generating function has closed form $\frac{1+x+x^2}{(1-x)^2}$ by multiplying $1 + x + x^2$ by the series for $1/(1-x)^2$.