PROBLEM 1. Suppose that $a \in \mathbb{R}^{\mathbb{N}}$ is a polynomial sequence of degree 4. Use the following table of differences to determine a formula for a_n.

a_n	0		0		4		12		72		$\cdots$
$\Delta[a]_n$		0		4		8		60		$\cdots$	
$\Delta^2[a]_n$			4		4		52		$\cdots$		
$\Delta^3[a]_n$				0		48		$\cdots$			
$\Delta^4[a]_n$					48		$\cdots$				

PROBLEM 2. With your group, choose a "random" polynomial p of degree at most 5. Prepare a table of the values $p(n)$ for $n = 0, 1, \ldots, 6$. Swap tables of values with another group and then reconstruct each others polynomials.

PROBLEM 3.

(i) For $r, n \geq 0$ define $a_{r,n} = \sum_{k=0}^{n} k^r$. Prove that $(a_{r,n})_{n=0}^{\infty}$ is a degree $r+1$ polynomial sequence.

(ii) Use a table of differences to determine a polynomial expression for
$$a_{3,n} = \sum_{k=0}^{n} k^3.$$

PROBLEM 4.

(i) Prove that $3 \mid n^3 + 2n$ for all $n \in \mathbb{N}$.

(ii) Suppose that f is a numerical polynomial of degree d. Prove that d divides $f(n)$ for all $n \in \mathbb{N}$ if and only if d divides $\Delta^k[f]_0$ for all $k \geq 0$.

(iii) What is the largest number dividing $n^5 - n$ for all $n \in \mathbb{N}$?