PROBLEM 1. When is $a \equiv b \pmod 2$? $a \equiv b \pmod 1$? $a \equiv b \pmod 0$?

PROBLEM 2. What are the last two digits of $99^{100000^{100000}+2021}$?

PROBLEM 3. Recall the equivalence relation from the mini-lecture: having fixed $n \in \mathbb{Z}$, for $a, b \in \mathbb{Z}$, we say $a \sim b$ if $a - b = kn$ for some $k \in \mathbb{Z}$. In other words, $a \sim b$ if and only if $a \equiv b \pmod n$. Take $n > 0$, for convenience.

(i) Show that $\sim$ is an equivalence relation.
(ii) State the division algorithm for integers a and n, and use it to determine the number of equivalence classes for $\sim$.

PROBLEM 4. Suppose $a \equiv a' \pmod n$ and $b \equiv b' \pmod n$.

(i) Prove that $a + b \equiv a' + b' \pmod n$.
(ii) Prove that $ab \equiv a'b' \pmod n$.

PROBLEM 5. (If you have extra time.) Let $V := \{0, 1, \cdots, n-1\}$ for some positive integer n, and fix $a \in V$. Let $G(a, n)$ be the directed graph with vertex set V and with an edge from b to c if $c = b + a \pmod n$. Draw this graph for various a and n, and try to deduce its general structure.