PROBLEM 1. Use the Euclidean algorithm to compute $\gcd(270, 192)$. Either back-solve or use the extended Euclidean algorithm to express $\gcd(270, 192)$ as an integer linear combination of 270 and 192, *i.e.*, find $s, t \in \mathbb{Z}$ such that

$$\gcd(270, 192) = 270s + 192t.$$

PROBLEM 2. Run the Euclidean algorithm when $a = 45$, $b = 16$. How is it related to the expression

$$\frac{45}{16} = 2 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{3}}}?$$

Come up with a general procedure by which the Euclidean algorithm produces *continued fraction* expressions for rational numbers of the form

$$\frac{a}{b} = x_1 + \cfrac{1}{x_2 + \cfrac{1}{x_3 + \cfrac{1}{x_4 + \cdots}}}$$

where the x_i are integers.

PROBLEM 3. The "rectangular" visualization of the Euclidean algorithm is a technique from ancient Greece known as *anthyphairesis*. It gives us a visual test for when the quotient of two real numbers x/y is a rational number.

(i) Thinking in terms of similar rectangles, argue that for x and y positive real numbers, $x/y = a/b$ for some $a, b \in \mathbb{N}$ if and only if the *anthyphairetic* dissection of an $x \times y$ rectangle terminates in a finite number of steps.

(ii) Use (i) to show that $\sqrt{2}/1$ is not a rational number.