PROBLEM 1. Find someone in your group who has taken calculus and have them assure you that

$$\frac{d}{dx}(x+1)^n = n(x+1)^{n-1}$$

and that

$$\frac{d}{dx}\sum_{k=0}^{n}\binom{n}{k}x^k = \sum_{k=1}^{n}k\binom{n}{k}x^{k-1}.$$

What identity results when you plug in $x = 1$? Can you find a proof that does not use calculus?

PROBLEM 2. Let

$$F = \{f\colon \{a,b,c,d\} \to [6] \mid f(a) \neq 1 \text{ or } f(b) \neq 2 \text{ or } f(c) \neq 3\}.$$

Determine $|F|$. (*Hint*: PIE.)

PROBLEM 3. Fix an integer $n \geq 1$ and let $V = 2^{[n]}$. Define a graph $G = (V, E)$ on the vertex set V such that for $A, B \in V$, $\{A, B\} \in E$ if and only if (1) $A \subseteq B$ or $B \subseteq A$ and (2) the larger of the two sets has exactly one more element than the smaller one.

(i) Draw G for $n = 1, 2$, and 3.
(ii) What is $|V|$?
(iii) Let $A \in V$ be a subset of $[n]$ of cardinality k. What is the degree of the vertex A? Explain.
(iv) There is a standard formula relating the degrees of the vertices to the number of edges. Use this formula to compute $|E|$.
(v) For which n does G have a closed Eulerian walk? Explain.

PROBLEM 4. Consider Joyal's bijection between $\mathcal{V}_n$, the set of vertebrates on vertex set $[n]$, and $[n]^{[n]}$, the set of functions $[n] \to [n]$.

(i) Which vertebrates correspond to bijections?
(ii) Which vertebrates correspond to constant functions?
(iii) Can you think of any other special classes of functions or vertebrates that are related by Joyal's bijection?

PROBLEM 5. List all of the Catalan structures you have encountered thus far and make a map illustrating how they are related.