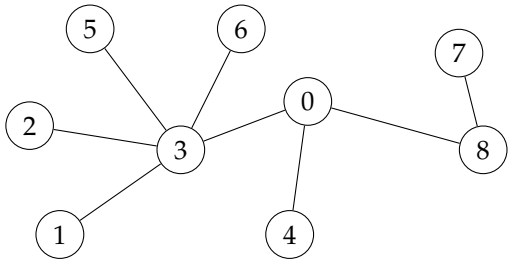
PROBLEM 1. Find the labeled Dyck path and corresponding labeled tree for the parking function $p = (3, 2, 5, 1, 2)$.

PROBLEM 2. Find the labeled Dyck path and corresponding labeled tree for the following parking functions: (i) $(1, 2, 3, 4, 5)$, and (ii) $(5, 4, 3, 2, 1)$.

PROBLEM 3. Let $p = (p_1, \ldots, p_n)$ be a parking function formed by permuting the entries of the increasing maximal parking function $(1, 2, \ldots, n)$. Describe the corresponding tree.

PROBLEM 4. Describe the parking function in bijection with the following labeled tree:



PROBLEM 5. Which trees correspond to increasing parking functions under this bijection? Note that this is a new Catalan structure! Directly describe a bijection between Dyck paths and this structure.

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

In the labeled Dyck path you constructed for Problem 4, forget the labels and just consider the Dyck path P, itself.

(i) Construct the balanced parenthesization B corresponding to P.
(ii) Is there a natural way to label B with the vertices of the tree from Problem 4, perhaps reflecting the labeling of P, that could lead to a bijection between labeled trees and labeled balanced parenthesizations in general?
(iii) One could ask the same question for any of the other Catalan structures we have studied. The next step might be to consider full binary trees.