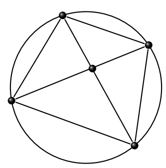
These problems assume you have viewed the video A Curious Pattern Indeed. Choose n points on a circle in the plane, and connect each pair of points with an edge. We are interested in counting the number of regions formed inside the circle.

PROBLEM 1. Does the number of regions depend on the positions of the points?

PROBLEM 2. Draw n vertices on a circle in the plane and connect each pair by an edge. Assume vertices are chosen so that the number of regions created in the circle is maximal. Let G_n be the graph formed by this drawing with the following provisos: we consider each intersection of a pair of lines in the drawing to be a vertex, and we include the circular arcs connecting consecutive vertices on the circle as edges. Check, for instance that G_4 has five vertices and twelve edges:



(i) For each of $n = 3, 4, 5, 6$, what is the vertex degree sequence of G_n. (Recall that the *vertex degree sequence* is the sorted list of $\deg(v)$ as v varies over the vertices of the graph.)
(ii) For general n, how many internal vertices does G_n have, i.e., vertices not on the circle? (Hint: quadrilaterals.)
(iii) What is the vertex degree sequence of G_n, in general .

PROBLEM 3. Use the vertex degrees from the previous problem to find a formula for the number of edges of G_n.

PROBLEM 4. Let G be any planar graph. Let V, E, and F be its sets of vertices, edges, and faces, respectively. We take the set of faces to included the single external (unbounded) face. Then Euler's formula says that

$$|V| - |E| + |F| = 2.$$

(i) Check Euler's formula for the graph G_4 pictured in Problem 2.
(ii) Combine your previous results to compute the number of (interior) regions in G_n.