PROBLEM 1. A *triangulation* of a convex n-gon is a collection of nonintersecting diagonals (line segments between non-adjacent vertices) that break the n-gon into triangles.

(i) Draw all triangulations of convex n-gons for $n = 3, 4, 5, 6$. Make a conjecture regarding the number of triangulations.
(ii) Prove your conjecture. (*Hint*: Label one side of the polygon as the base. Exactly one triangle in the triangulation includes the base edge. Use this triangle as the basis for a recursion.)

PROBLEM 2. From our reading, we know that full binary trees with $n + 1$ leaves and balanced parenthesizations of length $2n$ are counted by the Catalan number C_n. The reading also includes a description of a direct bijection between these two structures. Briefly, given a full binary tree, label the left edges with '(' on their left and ')' on their right. Start at the root of the tree and start walking down the leftwards edge; keep the tree on your left and record the labels as you pass them. The resulting is the balanced parenthesization corresponding to the binary tree.

Prove that the process described above works, i.e., that it provides a bijection. It is recommended that you follow these steps:

(i) Draw several full binary trees and produce the resulting balanced parenthesizations.
(ii) Prove that the resulting parenthesization is always balanced.
(iii) Describe an algorithm (or function) for turning a balanced parenthesization into a full binary tree which is inverse to the above assignment.

Challenge

Challenge problems are optional and should only be attempted after completing the previous problems.

Produce a direct bijection between triangulations of a convex n-gon and full binary trees with $n - 1$ leaves. Show that diagonal flips of edges in a triangulation correspond to tree rotations. (A *diagonal flip* transforms a quadrilateral ⧄ in a triangulation into ⧅.)