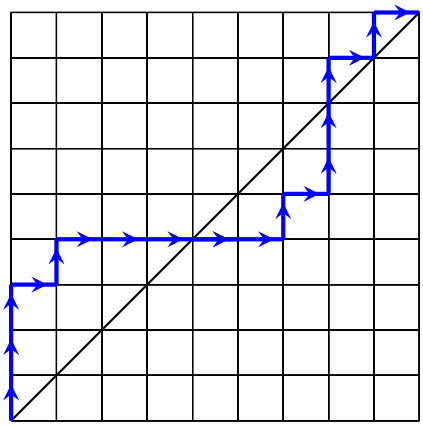
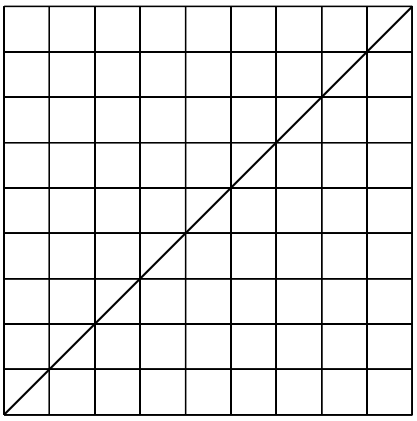
PROBLEM 1. Illustrate the bijection $E_i \to E_{i+1}$.

$BrAuC$

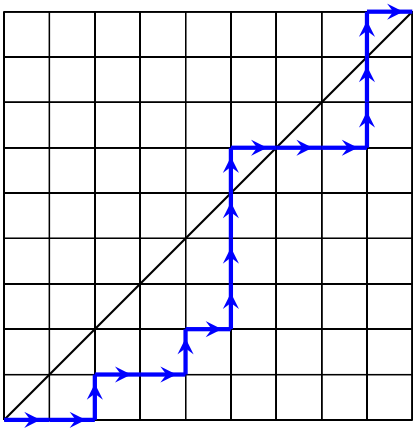


Exceedance = ☐

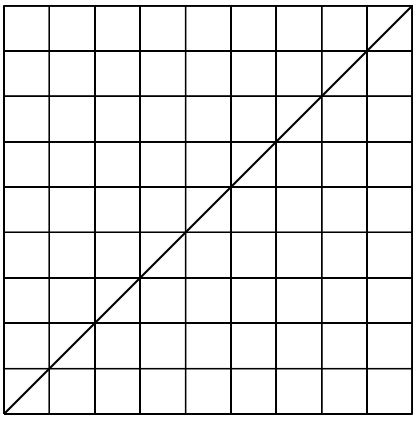
$AuBrC$



Exceedance = ☐



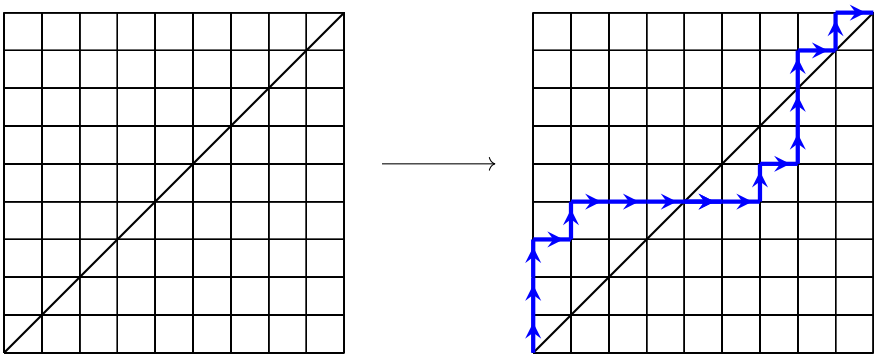
Exceedance = ☐



Exceedance = ☐

PROBLEM 2. Illustrate the inverse of the bijection $E_i \to E_{i+1}$. (Hint: something tricky occurs with A here.)

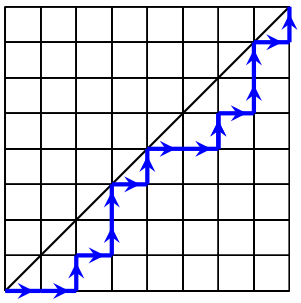
$BrAuC$ $\qquad\qquad$ $AuBrC$



Exceedance = ☐ $\qquad\qquad$ Exceedance = ☐

PROBLEM 3. In your reading, you saw that expressions consisting of n balanced parentheses $()$ are in bijection with Dyck paths of length $2n$, and, thus, the number of such expressions is the n-th Catalan number, C_n.

(i) Describe the bijection between Dyck paths and balanced parentheses, and apply it to the Dyck path below.



(ii) What is the Dyck path associated with $(((\,)\,)(\,)\,)$?

PROBLEM 4. Use the Catalan recurrence,

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{k=0}^{n} C_k C_{n-k} \text{ for } n \geq 0,$$

to compute the fifth Catalan number by hand.

PROBLEM 5. Explain the significance of the following sequence:

un, dos, tres, quatre, cinc, sis, set, vuit, nou, deu, ...